WENTWORTH-SMITH MATHEMATICAL SERIES

PLANE AND SOLID GEOMETRY

BY

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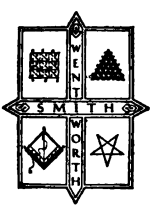

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PREFACE

Long after the death of Robert Recorde, England's first great writer of textbooks, the preface of a new edition of one of his works contained the appreciative statement that the book was "entail'd upon the People, ratified and sign'd by the approbation of Time." The language of this sentiment sounds quaint, but the noble tribute is as impressive to-day as when first put in print two hundred and fifty years ago.

With equal truth these words may be applied to the Geometry written by George A. Wentworth. For a generation it has been the leading textbook on the subject in America. It set a standard for usability that every subsequent writer upon geometry has tried to follow, and the number of pupils who have testified to its excellence has run well into the millions.

In undertaking to prepare a work to take the place of the Wentworth Geometry the authors have been guided by certain well-defined principles, based upon an extended investigation of the needs of the schools and upon a study of all that is best in the recent literature of the subject. The effects of these principles they feel should be summarized for the purpose of calling the attention of the wide circle of friends of the Wentworth-Smith series to the points of similarity and of difference in the two works.

1. Every effort has been made not only to preserve but to improve upon the simplicity of treatment, the clearness of expression, and the symmetry of page that characterized the successive editions of the Wentworth Geometry. It has been the purpose to prepare a book that should do even more than maintain the traditions this work has fostered.

2. The proofs have been given substantially in full, to the end that the pupil may always have before him a model for his independent treatment of the exercises.

3. The sequence of propositions has been improved in several respects, notably in the treatment of parallels.

4. To meet a general demand, the number of propositions has been decreased so as to include only the great basal theorems and problems. A little of the less important material has been placed in the Appendixes, to be used or not as circumstances demand.

5. The exercises, in some respects the most important part of a course in geometry, have been rendered more dignified in appearance and have been improved in content. The number of simple exercises has been greatly increased, while the difficult puzzle is much less in evidence than in most American textbooks. The exercises are systematically grouped, appearing in full pages, in large type, at frequent intervals. They are not all intended for one class, but are so numerous as to allow the teacher to make selections from year to year.

6. The work throughout has been made as concrete as is reasonable. Definitions have been postponed until they are actually needed, only well-recognized terms have been employed, the pupil is initiated at once into the practical use of the instruments, some of the reasons for studying geometry are early shown in an interesting way, application of geometry to practical cases in mensuration is provided for, and correlation is made with the algebra already studied.

The authors are indebted to many friends of the Wentworth-Smith series for assistance and encouragement in the labor of preparing this work, and they will welcome any further suggestions for improvement from any of their readers.

GEORGE WENTWORTH
DAVID EUGENE SMITH

CONTENTS

PLANE GEOMETRY

	PAGE
INTRODUCTION	1
BOOK I. RECTILINEAR FIGURES	25
Triangles	26
Parallel Lines	46
Triangles	51
Quadrilaterals	59
Polygons	68
Loci of Points	73
Methods of Proof	77
General Suggestions for Proving Theorems	84
Exercises	85
BOOK II. THE CIRCLE	93
Circles	93
Central Angles	94
Arcs and Chords	96
Secants and Tangents	102
Line of Centers	109
Measure of Angles	112
Problems of Construction	126
Solution of Problems	140
Exercises	141
BOOK III. PROPORTION. SIMILAR POLYGONS	151
Theory of Proportion	151
Proportional Lines	157
Similar Polygons	165
Numerical Properties of Figures	174
Problems of Construction	182
Exercises	187

PAGE

BOOK IV. AREAS OF POLYGONS 191
Areas of Polygons 192
Numerical Properties of Figures 204
Exercises 211
Problems of Construction 214
Exercises 223

BOOK V. REGULAR POLYGONS AND CIRCLES . . . 227
Regular Polygons and Circles 227
Problems of Construction 242
Problems of Computation 248
Exercises 250

APPENDIX TO PLANE GEOMETRY 261
Symmetry 261
Maxima and Minima 265

SOLID GEOMETRY

BOOK VI. LINES AND PLANES IN SPACE 273
Lines and Planes 273
Dihedral Angles 293
Polyhedral Angles 308
Exercises 314

BOOK VII. POLYHEDRONS, CYLINDERS, AND CONES . 317
Polyhedrons 317
Prisms 317
Parallelepipeds 322
Pyramids 337
Regular Polyhedrons 350
Cylinders 353
Cones 362
Exercises 376

BOOK VIII. THE SPHERE 381
Spheres 381
Plane Sections and Tangent Planes 383
Spherical Polygons 392
Measurement of Spherical Surfaces 410
Measurement of Spherical Solids 421
Exercises 424

PAGE

APPENDIX TO SOLID GEOMETRY 431

Polyhedrons 432

Spherical Segments 444

Exercises 446

RECREATIONS OF GEOMETRY 449

SUGGESTIONS AS TO BEGINNING DEMONSTRATIVE GEOMETRY 453

APPLICATIONS OF GEOMETRY 461

HISTORY OF GEOMETRY 465

TABLE OF FORMULAS 469

INDEX 471

SYMBOLS AND ABBREVIATIONS

$=$	equals, equal, equal to, is equal to, or is equivalent to.	Adj.	adjacent.
		Alt.	alternate.
		Ax.	axiom.
$>$	is greater than.	Const.	construction.
$<$	is less than.	Cor.	corollary.
$\parallel$	parallel.	Def.	definition.
$\perp$	perpendicular.	Ex.	exercise.
$\angle$	angle.	Ext.	exterior.
$\triangle$	triangle.	Fig.	figure.
$\square$	parallelogram.	Hyp.	hypothesis.
$\square$	rectangle.	Iden.	identity.
$\odot$	circle.	Int.	interior.
st.	straight.	Post.	postulate.
rt.	right.	Prob.	problem.
$\because$	since.	Prop.	proposition.
$\therefore$	therefore.	Sup.	supplementary.

These symbols take the plural form when necessary, as in the case of $\parallel$s, $\measuredangle$s, $\triangle$s, ⓢ.

The symbols $+$, $-$, $\times$, $\div$ are used as in algebra.

There is no generally accepted symbol for "is congruent to," and the words are used in this book. Some teachers use $\cong$ or $\cong$, and some use $\equiv$, but the sign of equality is more commonly employed, the context telling whether equality, equivalence, or congruence is to be understood.

Q. E. D. is an abbreviation that has long been used in geometry for the Latin words *quod erat demonstrandum*, "which was to be proved."

Q. E. F. stands for *quod erat faciendum*, "which was to be done."