MATH LEVEL 2
LESSON PLAN 3

FACTORING

Section 1: Exact Division & Factors

1. In exact division there is no remainder. Both Divisor and quotient are factors of the dividend.

   6 ÷ 2 = 3; **(no remainder)**
   Dividend Divisor Quotient

   2 x 3 = 6
   Factors

2. **Factors** of a number are two or more numbers, the product of which equals the given number.

   EXAMPLE: 2 and 3 are factors of 6, because 2 x 3 = 6.

   EXAMPLE: 3, 5 and 7 are factors of 105, because 3 x 5 x 7 = 105.

3. A **multiple** of a number is a product of which the number is a factor.

   EXAMPLE: 6 is a multiple of 3.

   EXAMPLE: 105 is a multiple of 7.

☺ EXERCISE

1. **State if the divisor is a factor of the dividend**

   (a) 9 ÷ 3 (b) 17 ÷ 5 (c) 20 ÷ 4 (d) 20 ÷ 3 (e) 28 ÷ 7

   Answer: (a) Yes (b No (c) Yes (d) No (e) Yes

2. **Find the missing factor**

   (a) 4 = 2 x __ (b) 6 = 2 x__ (c) 22 = 2 x__ (d) 16 = 4 x__ (e) 21 = 3 x __

   Answer: (a) 2 (b 3 (c) 11 (d) 4 (e) 7

3. **Write a pair of factors for the following numbers.**

   (a) 16 = __ x__ (b) 54 = __ x __(c) 36 = __ x __(d) 60 = __ x __

   Answer: Note: There could be more than one answer to the above problems.
   (a) 2 x 8 (b) 6 x 9 (c) 9 x 4 (d) 6 x 10

4. **State if the first number is a multiple of the second.**

   (a) 18 and 3 (b) 25 and 6 (c) 23 and 2 (d) 60 and 5 (e) 108 and 12

   Answer: (a) Yes (b No (c) No (d) No (e) Yes (f) Yes

Section 2: Composite & Prime Numbers

4. A number that can be factored into two or more smaller numbers is a **composite number**. The number **30** is a composite number.

   30 = 5 x 6

   In this case, the number **6** may be factored further, therefore **6** is also a composite number.

   6 = 3 x 2

   However, the number **5, 3 and 2** above cannot be factored into two smaller numbers. Therefore, they are called **prime numbers**.

☺ **EXERCISE**

State if the number is prime or composite

(a) 12 (b) 5 (c) 2 (d) 13 (e) 8 (f) 15 (g) 11 (h) 7 (i) 24 (j) 9 (k) 10

Answer: (a) composite (b) prime (c) prime (d) prime (e) composite (f) composite (g) prime (h) prime (i) composite (j) composite (k) composite

Section 3: Finding Prime Numbers

5. It is advantageous to know the prime numbers in advance. We may check the single-digit numbers as follows.

   0 → The number 0 represents no count and, therefore, it cannot be factored.
   0 is neither prime nor composite.

   1 → 1 is the unit of all numbers. It does not have two smaller factors (1 = 1 x 1).
   1 is neither prime nor composite.

   2 → 2 does not have two smaller factors (2 = 2 x 1).
   2 is the smallest prime number.

   All multiples of 2 shall be composite numbers. Therefore, all even number larger than 2 are composite numbers and not prime. 4, 6 and 8 are not prime numbers. We check odd numbers only from this point.

   3 → **3 is a single-digit prime number.**

   5 → **5 is a single-digit prime number.**

   7 → **7 is a single-digit prime number.**

   9 → We can factor 9 into two smaller numbers (9 = 3 x 3).

   Therefore, 9 is not a prime number because it is a multiple of 3. In general, the multiples of prime numbers are not prime numbers.

   The single-digit prime numbers are:

   2, 3, 5, and 7

6. We make a table (Table 1) to check for double-digit numbers prime numbers.

10	**11**	12	**13**	14	15	16	**17**	18	**19**
20	21	22	**23**	24	25	26	27	28	**29**
30	**31**	32	33	34	35	36	**37**	38	39
40	**41**	42	**43**	44	45	46	**47**	48	49
50	51	52	**53**	54	55	56	57	58	**59**
60	**61**	62	63	64	65	66	**67**	68	69
70	**71**	72	**73**	74	75	76	77	78	**79**
80	81	82	**83**	84	85	86	87	88	**89**
90	91	92	93	94	95	96	**97**	98	99

Table 1 – Double-digit Prime numbers

We then gray out those numbers that can be divided exactly by the single-digit prime numbers (2, 3, 5, and 7). The remaining numbers in bold are the two-digit prime numbers. These are: **11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89,** and **97**

7. Table 2 provides the prime numbers up to 1013:

2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	013	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	1009
59	139	233	337	439	557	653	769	883	1013

Table 2 – Prime Numbers up to 1013

Section 4: Prime Numbers for Factors

8. To test if a number is a prime number, we check to see if it can be divided exactly by the known prime numbers.

A number can be divided by 2 if it is an even number (last digit is 0, 2, 4, 6, or 8).

2 is a factor of **56** because the last digit is 6.
2 is a factor of **83,430** because the last digit is 0.

A number can be divided by 3 if the sum of its digits can be divided by 3.

3 is a factor of **897** because 8+9+7 = 24, and 2+4 = 6 (multiple of 3).
3 is a factor of **78,916,545** because 7+8+9+1+6+5+4+5 = 45, and 4+5 = 9.

A number can be divided by 5 if the last digit is 0 or 5.

5 is a factor of **735** because the last digit is 5.
5 is a factor of **37,230** because the last digit is 0.

A number can be divided by 7 if meets the following condition.

(1) Separate the last digit from the number. Then from the remaining number subtract the double of the last digit

For example, if the number is 38073, then separate it as 3807 and 3. Then calculate 3807 – (2 x3) = 3801

(2) Repeat this procedure as necessary.

For 3801, calculate 380 – (2 x 1) = 378

For 378, calculate 37 – (2 x 8) = 21

(3) If the final difference is 0 or divisible by 7, then 7 is a factor of the original number.

For the number 38073, the final difference is 21. Therefore, it can be divided by 7.

EXAMPLE: **Check if 60179 can be divided by 7.**
6017 – (2 x 9) = 5999
599 – (2 x 9) = 581
58 – (2 x 1) = 56 (divisible by 7)

Therefore, 60179 can be divided by 7.

☺ EXERCISE

Test to see if the following numbers can be divided by 2, 3, 5, and 7.

(a) 6585 (b) 9768 (c) 14154 (d) 4620 (e) 89712

Answer: (a) 3 and 5 (b) 2 and 3 (c) 2, 3 and 7 (d) 2, 3, 5 and 7 (e) 2, 3 and 7

Section 5: Prime Factors

9. A number may be expressed in terms of factors that are all prime numbers. This set of factors is unique for a number.

30 = 5 x 3 x 2 **(prime factors)**

16 = 2 x 2 x 2 x 2 **(prime factors)**

We may find the set of prime factors by continuing to factor the factors of a number until we end up with prime numbers. This generates a **factor tree** as shown below.

```
     24
     /\
   4 · 6
  /\   /\
2·2  2·3
```

We get, **24 = 2 x 2 x 2 x 3** **(prime factors)**

10. We may successively divide by prime numbers to find the prime factors of a number. It is easy and fast when we write the quotient below the dividend.

```
 2 | 35574
 3 | 17787
 7 |  5929
 7 |   847
11 |   121
        11
```

EXAMPLE: To find the prime factors of 35574, we check the smallest prime number 2 as the divisor. Then we check the next prime number, and so on. A prime number could be a divisor more than once. We continue dividing until the final quotient is a prime number.

35574 = 2 x 3 x 7 x 7 x 11 x 11

In the division method, one may use a calculator to check larger prime numbers. The point to stop checking is when the quotient becomes less than the divisor.

Example: Find the prime factors of **71996**.

```
 2 | 71996
 2 | 35998
41 | 17999
41 |   439
     10 R 29
```

After 2 we check successive prime numbers until we find 41 to be a prime factor. The quotient is 439. When we check 41 again.

439 ÷ 41 = 10 and a remainder

439 is not divisible by 41, and the resulting quotient is less than 41. So we stop here.

71996 = 2 x 2 x 41 x 439

☺ EXERCISE

1. Find the prime factors of the following numbers by factor tree

(a) 45 (b) 56 (c) 72 (d) 87 (e) 168 (f) 252 (g) 315 (h) 429 (i) 512 (j) 626

Answer: (a) 45=3x3x5 (b) 56=2x2x2x7 (c) 72=2x2x2x3x3 (d) 87=3x29 (e) 168=2x2x2x3x7 (f) 252= 2x2x3x3x7 (g) 315 =3x3x5x7 (h) 429=3x11x13 (i) 512=2x2x2x2x2x2x2x2x2 (j) 626=2x313

2. Find the prime factors of the following numbers by division as above

(a) 756	**(d) 2751**	**(g) 9768**	**(j) 89712**	**(m) 537152**
(b) 891	**(e) 4620**	**(h) 14157**	**(k) 333333**	**(n) 5666661**
(c) 1089	**(f) 6567**	**(i) 71996**	**(l) 405769**	**(o) 5056506**

Answer: (a) 756=2x2x3x3x3x7 (b) 891=3x3x3x 3x11 (c) 1089=3x3x11x11 (d) 2751=3x7x131 (e) 4620=2x2x3x5x7x11 (f) 6657=3x7x317 (g) 9768=2x2x2x3 x11x37 (h) 14157=3x3x11x11x13 (i) 71996=2x2x41x439 (j) 89712=2x2x2x2x3x3x7x89 (k) 333333=3x3x7x 11x13x37 (l) 405769=7x7x7x7x13x13 (m) 537152=2x2x2x2x2x2x11x763 (n) 5666661=3x3x7x11x13x17x37 (o) 5056506=2x3x3x3x3x7x7x7x13

Section 6: Common Prime Factors

11. To find the prime factors common to two or more numbers write the given numbers in a line. Divide by any prime number that will exactly divide all of them; divide the quotients in the same manner; and so continue to divide until no more common factors can be found.

EXAMPLE: What prime factors are common to 30 and 42?

2	**30,**	**42**
3	**15,**	**21**
	5,	**7**

The common prime factors are 2 and 3.

☺ **EXERCISE**

What prime factors are common to the following numbers?

(a) 60 and 90 (b) 56 and 88 (c) 63, 99 and 117 (d) 75, 125 and 175

Answer: (a) 2, 3 and 5 (b) 2, 2 and 2 (c) 3 and 3 (d) 5 and 5

Section 7: The Greatest Common Factor (GCF)

12. The greatest common factor (GCF) of two or more numbers is the biggest divisor they have in common. It contains all the prime factors common to the numbers, and no other factor.

13. To determine the GCF, find the prime factors common to the given numbers per Section 6. Multiply them together. The product will be the greatest common factor.

EXAMPLE: Find the GCF of 30 and 42?

2	**30,**	**42**
3	**15,**	**21**
	5,	**7**

GCF = 2 x 3 = **6**

EXAMPLE: Find the GCF of 42, 56 and 70.

2	**42,**	**56,**	**70**
7	**21,**	**28,**	**35**
	3,	**4,**	**5**

GCF = 2 x 7 = **14**

14. If the numbers are very large, the following method may be used to find the GCF:

Divide the greater number by the less, the divisor by the remainder, and so on, always dividing the last divisor by the last remainder, until nothing remains. The last divisor will be the greatest common divisor.

EXAMPLE: Find the GCF of 8427 and 10017

10017 ÷ 8427	=	1	remainder 1590
8427 ÷ 1590	=	5	remainder 477
1590 ÷ 477	=	3	remainder 159
477 ÷ 159	=	3	**no remainder**

The GCF is 159.

To find the GCF of more than two numbers by this method, first find the GCF of two of them, then of that common factor and one of the remaining numbers, and so on for all the numbers; the last common factor will be the GCF of all the numbers.

15. Here is an example of a real problem that requires the calculation of GCF:

Suppose you want to find the biggest size of the barrel in which you can store 20391 gallons of beer and 49287 gallons of wine without mixing them together, and no empty space left. The answer would be the GCF of these two amounts.

The Greatest Common Factor (GCF) of the numbers 20391 and 49287 is 21. Therefore the *biggest* size of the barrel would be 21 gallons.

☺ EXERCISE

Find the Greatest Common Factor (GCF) of the following numbers.

(a) 120 and 216 (b) 76 and 133 (c) 248 and 465 (d) 96, 144 and 216

Answer: (a) 24 (b) 19 (c) 2, 3 and 7 (d) 31 (e) 24

Section 8: The Least Common Multiple (LCM)

16. The least common multiple (LCM) is the smallest multiple common to two numbers. It contains all the prime factors of each number and no other factor.

Thus, the LCM of 12 and 18 is 36. 36 = 2 x 2 x 3 x 3. It must contain all these factors , else it would not contain 12 = 2 x 2 x 3, and 18 = 2 x 3 x 3. It must not contain no other factor, else it would not be the least common multiple.

17. To determine the LCM, write the given numbers in a line. Divide by any prime number that will exactly divide two or more of them. Write the quotients and undivided numbers in a line beneath. Divide these numbers in the same manner, and so continue the operation until a line is reached in which no two numbers have common factors. Then the product of the divisors and the numbers in the last line will be the least common multiple.

EXAMPLE: Find the LCM of 4, 6, 9 and 12.

2	4,	6,	9,	12	(2 divides into 3 of the numbers)
3	2,	3,	9,	6	(3 divides into 3 of the numbers)
2	2,	1,	3,	2	(2 divides into 2 of the numbers)
	1,	1,	3,	1	

LCM = 2 x 3 x 2 x 3 = 36

This LCM contains the factors of 4 = 2 x 2; 6 = 2 x 3; 9 = 3 x 3; and 12 = 2 x 2 x 3, and no other factor.

EXAMPLE: Find the LCM of 36, 40, 45, and 50.

2	36,	40,	45,	50	(2 divides into 3 of the numbers)
5	18,	20,	45,	25	(5 divides into 3 of the numbers)
3	18,	4,	9,	5	(3 divides into 2 of the numbers)
3	6,	4,	3,	5	(3 divides into 2 of the numbers)
2	2,	4,	1,	5	(2 divides into 2 of the numbers)
	1,	2,	1,	5	

LCM = 2 x 5 x 3 x 3 x 2 x 2 x 5 = 1800

☺ EXERCISE

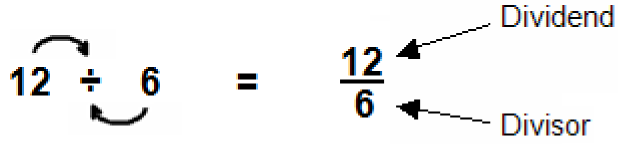
Find the LCM (Least Common Multiple) of the following:

(a) 4 and 9 (c) 14 and 42 (e) 6, 15 and 18 (g) 26, 33, 39 and 44
(b) 6 and 9 (d) 36 and 60 (f) 6, 13 and 26

Answer: (a) 36 (b) 18 (c) 42 (d) 180 (e) 90 (f) 78 (g) 1716

Section 9: Division by Factoring

18. We may write the division as “dividend over divisor”.



We then replace the dividend and divisor by their prime factors. For example,

$$1092 \div 182 = \frac{1092}{182} = \frac{2 \times 2 \times 3 \times 7 \times 13}{2 \times 7 \times 13}$$

We then cancel out the same factors above and below the line.

$$= \frac{\cancel{2} \times 2 \times 3 \times \cancel{7} \times \cancel{13}}{\cancel{2} \times \cancel{7} \times \cancel{13}}$$

This does not change the value because a number divided by itself is 1, and a number multiplied by 1 is the same number. What remains then gives us the quotient of the division.

$$= \frac{2 \times 3}{1} = 6$$

19. We get the same result when we divide the two numbers above and below the line by the same factor until the bottom number becomes 1.

$$1092 \div 182 = \frac{\cancel{1092}^{546}}{\cancel{182}_{91}} \quad \text{(divide up and down by 2)}$$

$$= \frac{\cancel{546}^{78}}{\cancel{91}_{13}} \quad \text{(divide up and down by 7)}$$

$$= \frac{\cancel{78}^{6}}{\cancel{13}_{1}} \quad \text{(divide up and down by 13)}$$

$$= 6$$

20. When multiplication and division occur together, we write the dividends above the line, and divisors below the line, as factors. We then cancel out the common factors from top and bottom, as shown below.

(a) $6 \times 8 \div 4 \div 3 = \dfrac{6 \times 8}{4 \times 3} = \dfrac{\overset{2}{\cancel{6}} \times \overset{2}{\cancel{8}}}{\underset{1}{\cancel{4}} \times \underset{1}{\cancel{3}}} = 4$

(b) $40 \div 5 \times 8 \div 4 = \dfrac{40 \times 8}{5 \times 4} = \dfrac{\overset{8}{\cancel{40}} \times \overset{2}{\cancel{8}}}{\underset{1}{\cancel{5}} \times \underset{1}{\cancel{4}}} = 16$

(c) $18 \div 7 \div 9 \times 14 = \dfrac{18 \times 14}{7 \times 9} = \dfrac{\overset{2}{\cancel{18}} \times \overset{2}{\cancel{14}}}{\underset{1}{\cancel{7}} \times \underset{1}{\cancel{9}}} = 4$

☺ EXERCISE

1. Divide by canceling the common factors

(a) 36 ÷ 12 **(d) 504 ÷ 36** **(g) 189 ÷ 21** **(j) 640 ÷ 40** **(m) 806 ÷ 26**
(b) 98 ÷ 14 **(e) 980 ÷ 28** **(h) 350 ÷ 14** **(k) 783 ÷ 27** **(n) 966 ÷ 42**
(c) 125 ÷ 25 **(f) 270 ÷ 18** **(i) 272 ÷ 16** **(l) 544 ÷ 32** **(o) 3885 ÷ 105**

Answer: (a) 3 (b) 7 (c) 5 (d) 14 (e) 35 (f) 15 (g) 9 (h) 25 (i) 17 (j) 16 (k) 29 (l) 17 (m) 31 (n) 23 (o) 37

2. Compute the following.

(a) 6 x 16 x 5 ÷ 5 ÷ 6 ÷ 8 **(d) 8 x 23 x 15 ÷ 5 ÷ 23 ÷ 8**
(b) 21 ÷ 8 x 2 ÷ 21 x 8 **(e) 17 ÷ 8 x 5 ÷ 17 x 8**
(c) 13 ÷ 2 ÷ 5 ÷ 13 x 10 **(f) 24 ÷ 8 ÷ 2 ÷ 24 x 32**

Answer: (a) 2 (b) 2 (c) 1 (d) 3 (e) 5 (f) 2

☺ L2 Lesson Plan 3: Check your Understanding

1. **What is the smallest prime number and why?**
2. **Reduce the number 4620 to its prime factors.**
3. **Write a list of even prime numbers.**
4. **Write the prime numbers between 100 and 120.**
5. **Use calculator to find the smallest 4-digit prime number.**
6. **Divide 966 by 42 using factoring.**
7. **Find the GCF of 1472 and 1792.**
8. **Find the LCM of 12, 28, and 42.**

Check your answers against the answers given below.

Answer:

1) The smallest prime number is 2. The number 0 represents no count and, therefore, it cannot be factored. 1 does not have a pair of two smaller factors.
2) 4620=2x2x3x5x7x11
3) The only even prime number is 2 because all other even numbers are multiples of 2. Therefore, all prime numbers other than 2 are odd.
4) 3) 101, 103, 107, 109, 113 because these numbers cannot be divided exactly by 2, 3, 5 and 7 or the next prime number 11.
5) 1009. The smallest 4-digit number is 1000. The square root of this number is less than 32. The prime numbers up to 32 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 and 31. Start checking these prime numbers as factors of 1000 onwards. 1009 is the first number, which does not have any of these as a factor. Therefore, 1009 is a prime number.
6) 23
7) 64
8) 84