

MATH LEVEL 2
LESSON PLAN 2

ARITHMETIC EXPRESSION

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Section 1: Expression & Terms

1. An arithmetic expression is made up of numbers joined by addition (+), subtraction (-), multiplication (x) and division (÷). Here is an example of such expression.

$$4 \times 7 - 27 + 8 \div 2 - 9$$

2. An expression is made up of terms separated by plus (+) and minus (-). The multiplications (x) and divisions (÷) are part of the term. In the following expressions the terms are underlined.

$$\underline{4 \times 7} - \underline{27} + \underline{8 \div 2} - \underline{9} \quad (4 \text{ terms})$$

3. Here are some examples of arithmetic expressions. Write these expressions down on a piece of paper. Circle the (+) and (-), and then underline the terms separated by them. Verify the number of terms.

$$8 \times 3 + 2 \quad (2 \text{ terms})$$

$$12 \times 14 + 6 \div 4 - 2 \times 18 + 9 + 5 \quad (5 \text{ terms})$$

$$12 \times 14 \div 6 \div 4 + 2 \times 18 \div 9 \quad (2 \text{ terms})$$

$$12 \times 7 \div 6 \div 4 \times 2 \times 18 \div 9 \quad (1 \text{ term})$$

 **EXERCISE**

Identify how many terms there are in each expression.

(a) $6 \times 16 \times 5 \div 5 \div 6 \div 8$

(d) $3 + 5 \times 4 - 8 \div 4 \times 3 + 7$

(b) $21 \div 8 + 2 \div 21 - 8$

(e) $5 \times 9 \div 3 + 32 \div 2 \div 2 \div 2$

(c) $13 - 2 + 5 - 13 + 10$

(f) $36 - 6 - 6 - 6 - 6 - 6 - 6$

Answer: (a) 1 term (b) 3 terms (c) 5 terms (d) 4 terms (e) 2 terms (f) 7 terms

Section 2: Reducing Expressions

4. To reduce an expression, we first reduce each individual term to a number, and then reduce the whole expression to a number.

Example: Reduce the expression “ $4 \times 7 - 27 + 8 \div 2 - 9$ ”

$$4 \times 7 - 27 + 8 \div 2 - 9 = \underline{4 \times 7} - \underline{27} + \underline{8 \div 2} - \underline{9}$$

$$= 28 - 27 + 4 - 9$$

$$= -4$$

5. Complex terms are made up of multiplication (x) and division (÷).

Complex term: $6 \times 2 \div 3$

In this term the x and ÷ apply to the number to their right.

The sign \times applies to 2.

The sign \div applies to 3.

The sign that applies to 6 is not shown but it is \times , because

$$6 \times 2 \div 3 = 1 \times 6 \times 2 \div 3$$

6. To reduce a complex term we move multipliers to the top and divisors to the bottom of a line as follows.

$$8 \div 7 \times 5 \div 8 \times 7 = \frac{8 \times 5 \times 7}{7 \times 8}$$

We cancel out the same number above and below the line because they reduce to 1.

$$\frac{8 \times 5 \times 7}{7 \times 8} = \frac{\cancel{8} \times 5 \times \cancel{7}}{\cancel{7} \times \cancel{8}} = 5$$

7. In general we divide the top numbers by the bottom numbers.

$$22 \div 7 \times 14 \div 11 = \frac{\overset{2}{\cancel{22}} \times \overset{2}{\cancel{14}}}{\underset{1}{\cancel{7}} \times \underset{1}{\cancel{11}}} = 2 \times 2 = 4$$

8. Here is example of reducing terms first and then the expression.

$$\begin{aligned} \text{Expression: } & 13 - 2 \times 5 + 13 + 10 \times 24 \div 16 + 5 \\ & = \underline{13} - \underline{2 \times 5} + \underline{13} + \underline{(10 \times 24 \div 16)} + \underline{5} \\ & = 13 - 10 + 13 + 15 + 5 \\ & = 36 \end{aligned}$$

9. The expression within parentheses represents a single number: Therefore, parentheses are reduced first to a single number.

$$\begin{aligned} \text{Expression: } & (16 + 6) \div 11 + 5 \\ & = 22 \div 11 + 5 \\ & = \underline{22 \div 11} + \underline{5} \\ & = 2 + 5 \\ & = 7 \end{aligned}$$

10. Here is another example of reducing terms first and then the expression.

$$\begin{aligned} \text{Expression: } & 4 \times 7 - 9 \times 3 + 8 \div 2 \\ & = \underline{4 \times 7} - \underline{9 \times 3} + \underline{8 \div 2} \\ & = 28 - 27 + 4 \\ & = 5 \end{aligned}$$

😊 EXERCISE

1. Find the value of the following terms.

$$(a) 7 \div 3 \times 6 \quad (b) 9 \div 2 \times 4 \quad (c) 30 \div 5 \div 2 \div 3 \times 5 \quad (d) 24 \div 2 \times 4 \div 3 \times 2 \div 4$$

Answer: (a) 14 (b) 18 (c) 5 (d) 8

😊 EXERCISE

Reduce the following complex terms

- (a) $6 \times 16 \times 5 \div 5 \div 6 \div 8$ (d) $8 \times 23 \times 15 \div 5 \div 23 \div 8$
(b) $21 \div 8 \times 2 \div 21 \times 8$ (e) $17 \div 8 \times 5 \div 17 \times 8$
(c) $13 \div 2 \div 5 \div 13 \times 10$ (f) $24 \div 8 \div 2 \div 24 \times 32$

Answer: (a) 2 (b) 2 (c) 1 (d) 3 (e) 5 (f) 2

😊 EXERCISE

Reduce the following expressions

- (a) $8 + 2 \times 4$ (d) $(3 + 8) \times 5$ (g) $4 \times 3 + 2 \times 5$
(b) $8 \times 2 + 4$ (e) $6 + 5 \times 3$ (h) $4 + 3 \times 2 + 5$
(c) $3 + 8 \times 5$ (f) $6 \times (5 + 3)$ (i) $4 + 3 \times (2 + 5)$

Answer: (a) 16 (b) 20 (c) 43 (d) 55 (e) 21 (f) 48 (g) 22 (h) 15 (i) 25

Section 3: Order of Operations

11. The logical order of operations is as follows:

The most basic operation is COUNTING

Order 0: COUNTING

ADDITION is “Repeated Counting”. The reverse of addition is SUBTRACTION. We may resolve addition and subtraction at the same level.

Order 1: ADDITION & SUBTRACTION

MULTIPLICATION is “Repeated Addition”. The reverse of multiplication is DIVISION. We may resolve multiplication and division at the same level.

Order 2: MULTIPLICATION & DIVISION

Expressions in parentheses are treated as a single number. Therefore, such expressions are resolved within the isolation of parentheses.

We, therefore resolve an expression in the following order

- a) **Operations in parentheses first**
- b) **Multiplication and division next**
- c) **Addition and subtraction after that**

12. The order of operations is automatically taken into account when we reduce the terms first and then reduce the expression.

13. Schools teach the order of operations as PEMDAS (Please Excuse My Dear Aunt Sally) This “order” results in error when multiplication is carried out before division, and addition is carried out before subtraction without understanding the signs of numbers.

$$6 \div 2 \times 3 = 9 \quad \text{and} \quad \text{NOT} \quad 1 \quad \text{as per PEMDAS}$$

$$8 - 5 + 3 = 6 \quad \text{and} \quad \text{NOT} \quad 0 \quad \text{as per PEMDAS}$$

☺ **EXERCISE**

Reduce the following expressions to a number

(a) $6 \times 6 - 5 \times 6 + 3 \div 3 + 3 \div 3 - 4$

(b) $18 \div 9 + 55 \div 11 - 21 \div 3 + 2 + 1$

(c) $8 \div 4 \times 3 - 4 \times 4 \div 2 + 6 \div 15 \times 5$

(d) $3 + 5 \times 4 - 8 \div 4 \times 3 + 7 - 12 \div 3$

(e) $21 \div 3 - 21 \div 7 + 8 \times 3 \div 12 + 13$

(f) $13 - 2 \times 5 + 13 + 10 \times 24 \div 16 + 5$

Answer: (a) 4 (b) 3 (c) 0 (d) 20 (e) 19 (f) 36

☺ **ADDITIONAL EXERCISE**

Reduce the following expressions to a number

(a) $8 \times 3 \div 12$ = _____

(b) $13 - 16 \div 8$ = _____

(c) $20 \times 3 \div 5 \div 6$ = _____

(d) $3 \times 4 + 9 \div 3$ = _____

(e) $60 \div 4 \div 3 \div 5$ = _____

(f) $2 \times 6 \times 7 \times 1 \div 14$ = _____

(g) $21 \times 6 \div 7 \div 3$ = _____

(h) $5 \times 6 \div 10 + 5 \times 8 \div 20$ = _____

(i) $5 \div 6 \times 5 \times 12 \div 25$ = _____

(j) $8 \times 6 + 12 \times 5 \div 3 + 12$ = _____

(k) $21 \times 35 \times 24 \div 49 \div 15 \div 12$ = _____

(l) $128 \div 2 \div 2 \div 2 \div 2 \div 2 \div 2 \div 2$ = _____

(m) $56 \times 54 \div 6 \times 5 \div 8 \div 9$ = _____

(n) $6 \times 6 - 6 \times 6 + 3 \div 3 - 3 \div 3 + 4$ = _____

(o) $18 \div 11 \times 55 \div 14 \times 21 \div 9$ = _____

(p) $8 \div 6 \times 3 + 4 \times 2 - 2 \times 6 \div 4$ = _____

Answer: (a) 2 (b) 11 (c) 2 (d) 15 (e) 1 (f) 6 (g) 6 (h) 5 (i) 2 (j) 80 (k) 2 (l) 1 (m) 35 (n) 4 (o) 15 (p) 9

Section 4: Word Problems

14. Translate a word problem to an arithmetic expression while solving the terms first.

Example question

"How much more will 143 horses cost, at \$77 each, than 367 cows, at \$30 each?"

Answer

$143 \times 77 - 367 \times 30 = 11011 - 11010 = \1

☺ **L2 Lesson Plan 2: Check your Understanding**

1. Explain term and expression.
2. Isolate the terms in parentheses.
(a) $6 \times 6 - 5 \times 6 + 3 \div 3 + 3 \div 3 - 4$
(b) $8 \div 4 \times 3 - 4 \times 4 \div 2 + 6 \div 15 \times 5$
3. Reduce the following expressions to a single number?
(a) $4 + 3 \times (2 + 5)$
(b) $13 - 2 \times 5 + 13 + 10 \times 24 \div 16 + 5$

Check your answers against the answers given below.

Answer

- 1) Numbers joined together by addition, subtraction, multiplication and division generate an arithmetic expression. In an expression, addition (+) and subtraction (-) separate the terms. Within terms we have multiplication (x) and division (\div). A term can simply be a number.
- 2) (a) $(6 \times 6) - (5 \times 6) + (3 \div 3) + (3 \div 3) - 4$
(b) $(8 \div 4 \times 3) - (4 \times 4 \div 2) + (6 \div 15 \times 5)$
- 3) (a) 25
(b) 36