

MATH LEVEL 2
LESSON PLAN 1
INTEGERS

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Section 1: Zero & Integers

1. Zero means “absence of quantity”. Therefore, a quantity is measured from zero.

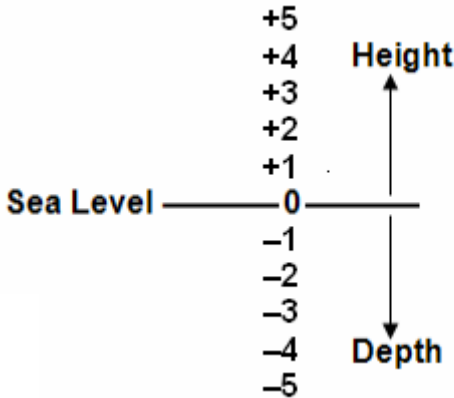
A quantity may be measured in two different directions, such as, the distance above the sea level (height) and below the sea level (depth).

The distance above the sea level becomes $0 + 1, 0 + 2, 0 + 3$, etc. This form is shortened as **+1, +2, +3, and so on.**

The distance below the sea level becomes $0 - 1, 0 - 2, 0 - 3$, etc. This form shortened as **-1, -2, -3, and so on.**

The shortened forms above are called positive and negative integers. These integers may be represented on a scale as shown below. They continue to infinity in both directions.

INTEGERS are whole numbers that are counted indefinitely from zero in both positive and negative directions on a number line.



2. Zero “0” may be placed anywhere on this scale as the reference point. For example, on the Centigrade temperature scale we set to zero the temperature at which pure ice melts. But on Fahrenheit temperature scale we set to zero the temperature at which “salt water” ice melts.

Another example is the calendar on which zero for BC is the birth year of Christ, and the zero for AD is the death year of Christ. The period of Christ’s life is not represented on this Calendar.

3. Integers increase to the right and decrease to the left on the number line. Therefore, a number on right is greater than a number on left.

	+3	>	-5;	“>” is a sign for “greater than”
And,	-1	<	+1;	“<” is a sign for “less than”
We have,	-2	>	-4;	“>” or “<” opens toward the larger number

4. An integer is made up of a sign and a number. The sign is part of the integer. The integers may be concatenated (linked together as in a chain). The linked integers may be treated as part of an “arithmetic expression” made up of addition and subtraction as follows

$$+15 +5 -10 = 0 + 15 + 5 - 10 = 15 + 5 - 10$$

Therefore, if an arithmetic expression is made up of addition (+) and subtraction (–) only, and the sign of the beginning number is not shown, then it is taken as positive.

😊 EXERCISE

1. Give another example of zero as a reference point

Answer: We measure our height from the surface we are standing on as zero..

2. Express the following as integers on a scale.

- (a) A tree 50 feet high (c) A temperature 20 °C warmer than freezing water
 (b) A ditch 4 feet deep (d) A shortage of \$25 in bank account

Answer: (a) +50 (b) -4 (c) +20 (d) -25

3. Express the following addition and subtraction as integers

- (a) 0 – 5 (c) 0 + 13 (e) 0 – 37
 (b) 0 + 5 (d) 0 – 13 (f) 0 + 37

Answer: (a) -5 (b) +5 (c) +13 (d) -13 (e) -37 (f) +37

4. Place the correct symbol, > or <, between the two numbers.

- (a) 3 5 (c) 7 4 (e) -2 -5 (g) -6 -1
 (b) -2 13 (d) 3 -7 (f) 5 -6 (h) -11 -8

Answer: (a) < (b) < (c) > (d) > (e) > (f) > (g) < (h) <

5. Write the numbers from the following expression as integers arranged from smallest to the largest

$$9 - 3 + 5 - 6 - 1 + 8 - 4 + 7 - 2$$

Answer: -6, -4, -3, -2, -1, +5, +7, +8, +9

Section 2: Combining Integers

5. When we combine integers, it is like adding and subtracting them as follows. Think of a number line. Start from 0. For + move to the right; for – move to the left, as many places as the integer. The last number you end up with is the combined value.

$$\begin{aligned} +8 -3 &= 0 + 8 - 3 &= 0 + 5 &= +5 \\ -8 +3 &= 0 - 8 + 3 &= 0 - 5 &= -5 \\ +8 +3 &= 0 + 8 + 3 &= 0 + 11 &= +11 \\ -8 -3 &= 0 - 8 - 3 &= 0 - 11 &= -11 \end{aligned}$$

6. The sign explains the operation on the number to its right. The number may be moved around with its sign. This does not change the value of the expression. For example, we can move around “- 10” in the following expression without changing the final outcome.

$$15 + 5 - 10 = 15 - 10 + 5 = 0 - 10 + 15 + 5 = 10$$

The sign explains the operation on the number to which it belongs. In the first example below 3 cannot be added to 9. It may only be subtracted.

Ex. 1: $12 - 3 + 9$ is not 0. The correct value is 18.

Ex. 2: $2 - 7 + 11$ is $2 + 11 - 7 = 2 + 4 = 6$

7. In an expression consisting of addition and subtraction, the same number with opposite signs may be canceled. This is because a number subtracted from itself is 0.

$$\begin{aligned} 8 - 5 + 3 - 4 - 3 + 7 + 4 - 8 &= \cancel{8} - 5 + \cancel{3} - \cancel{4} - \cancel{3} + 7 + \cancel{4} - \cancel{8} \\ &= 0 - 5 + 7 \\ &= 2 \end{aligned}$$

8. Taking away the negative numbers individually is the same as taking them away together as a sum.

$$25 - 6 - 5 - 9 = 25 - (6 + 5 + 9) = 25 - 20 = 5$$

9. We may collect positive and negative integers separately as two sums, and then combine them.

$$\begin{aligned} 9 - 3 + 5 - 6 - 1 &= (9 + 5) - (3 + 6 + 1) \\ &= 14 - 10 \\ &= 4 \end{aligned}$$

😊 EXERCISE

1. Combine the following integers.

$$\begin{array}{llll} \text{(a)} -8 + 7 = \underline{\quad} & \text{(c)} +9 - 4 = \underline{\quad} & \text{(e)} -7 - 2 = \underline{\quad} & \text{(g)} +8 + 6 = \underline{\quad} \\ \text{(b)} -2 - 8 = \underline{\quad} & \text{(d)} +5 + 4 = \underline{\quad} & \text{(f)} -7 + 15 = \underline{\quad} & \text{(h)} +11 - 27 = \underline{\quad} \end{array}$$

Answer: (a) -1 (b) -10 (c) +5 (d) +9 (e) -9 (f) +8 (g) +14 (h) -16

2. Find the value of the following expressions.

$$\text{(a)} 8 - 3 + 5 \quad \text{(b)} 5 - 8 + 10$$

Answer: (a) 10 (b) 7

3. Subtract the following by grouping the negative numbers

$$\text{(a)} 30 - 15 - 10 \quad \text{(b)} 17 - 5 - 5 - 5$$

Answer: (a) $30 - 25 = 5$ (b) $17 - 15 = 2$

4. Combine the following integers.

$$\begin{array}{l} \text{(a)} +4 - 16 - 9 + 25 - 3 - 11 + 7 \\ \text{(b)} +8 - 4 - 14 + 18 - 5 + 7 + 2 \end{array}$$

Answer: (a) -3 (b) +12

5. Reduce the following expressions

$$\begin{array}{lll} \text{(a)} 7 - 4 + 2 & \text{(d)} 9 - 3 - 4 + 1 & \text{(g)} 3 - 5 - 2 + 9 - 3 - 4 - 5 + 7 \\ \text{(b)} 7 + 5 - 8 & \text{(e)} 4 - 5 - 7 + 13 & \text{(h)} 9 - 3 - 6 - 2 + 8 - 3 + 7 - 2 \\ \text{(c)} 2 + 6 - 5 & \text{(f)} 6 - 9 + 15 - 5 & \text{(i)} 7 - 6 - 5 - 11 + 5 + 9 - 2 + 7 \end{array}$$

Answer: (a) 5 (b) 4 (c) 3 (d) 3 (e) 5 (f) 7 (g) 0 (h) 8 (i) 4

Section 3: Adding & Subtracting Integers

10. The + sign means “positive”. It does not change the sign of the integer.

$$\begin{array}{l} + (+5) = +5 \\ + (-3) = -3 \end{array}$$

The – sign means “negative”. It reverses the sign of the integer.

$$\begin{array}{l} - (+5) = -5 \\ - (-3) = +3 \end{array}$$

11. We may say that when the consecutive signs are the same, the integer is +ve.

$$\begin{array}{l} + (+5) = +5 \\ - (-3) = +3 \end{array}$$

We may say that when the consecutive signs are different, the integer is –ve.

$$\begin{array}{l} + (-5) = -5 \\ - (+3) = -3 \end{array}$$

12. When adding integers, first reduce the consecutive signs, and then combine the resulting integers per Section 2.

$$\begin{array}{llll} (+3) + (+5) & = & +3 +5 & = & +8 \\ (-3) + (-5) & = & -3 -5 & = & -8 \\ (+3) + (-5) & = & +3 -5 & = & -2 \\ (-3) + (+5) & = & -3 +5 & = & +2 \end{array}$$

13. When subtracting integers, first reduce the consecutive signs, and then combine the resulting integers per Section 2.

$$\begin{array}{llll} (+3) - (+5) & = & +3 -5 & = & -2 \\ (-3) - (-5) & = & -3 +5 & = & +2 \\ (+3) - (-5) & = & +3 +5 & = & +8 \\ (-3) - (+5) & = & -3 -5 & = & -8 \end{array}$$

14. We may simplify an expression made up of integers as follows.

$$\begin{aligned} +3 + (-5) + (+2) - (-9) + (-3) &= +\cancel{3} -5 +2 +9 -\cancel{3} \\ &= (2 + 9) - 5 \\ &= 6 \end{aligned}$$

☺ EXERCISE

1. Place the resulting sign for the integer

$$\begin{array}{llll} \text{(a) } - (+7) = \square 7 & \text{(c) } + (-4) = \square 4 & \text{(e) } - (-2) = \square 2 & \text{(g) } + (+6) = \square 6 \\ \text{(b) } - (-8) = \square 8 & \text{(d) } + (+1) = \square 1 & \text{(f) } - (+15) = \square 15 & \text{(h) } + (-27) = \square 27 \end{array}$$

Answer: (a) -7 (b) +8 (c) -4 (d) +1 (e) +2 (f) -15 (g) +6 (h) -27

2. Add and subtract the following integers.

$$\begin{array}{llll} \text{(a) } -8 + (-7) = \square & \text{(c) } +9 - (-4) = \square & \text{(e) } -7 - (-2) = \square & \text{(g) } +8 + (-6) = \square \\ \text{(b) } -2 - (-8) = \square & \text{(d) } -5 + (+4) = \square & \text{(f) } +7 - (+15) = \square & \text{(h) } +11 + (-27) = \square \end{array}$$

Answer: (a) -15 (b) +6 (c) +13 (d) -1 (e) -5 (f) -8 (g) +2 (h) -16

3. Solve the following mixed operations.

$$\begin{array}{ll} \text{(a) } -8 + (-7) + (+5) - (-4) - (+5) = \square & \text{(c) } -7 - (-2) + (-3) + (-2) - (-3) = \square \\ \text{(b) } -2 - (-8) - (+7) + (-6) + (+7) = \square & \text{(d) } +7 - (+15) + (+11) - (-27) + (-17) = \square \end{array}$$

Answer: (a) -11 (b) 0 (c) -7 (d) +13

Section 4: Multiplying & Dividing Integers

15. In multiplication and division of integers, we may treat the signs as consecutive signs.

$$\begin{array}{llll} (+12) \times (+4) & = & (+)(+)(12 \times 4) & = & +48 \\ (+12) \times (-4) & = & (+)(-)(12 \times 4) & = & -48 \\ (-12) \times (-4) & = & (-)(-)(12 \times 4) & = & +48 \\ (-12) \times (+4) & = & (-)(+)(12 \times 4) & = & -48 \\ (+12) \div (+4) & = & (+)(+)(12 \div 4) & = & +3 \\ (+12) \div (-4) & = & (+)(-)(12 \div 4) & = & -3 \\ (-12) \div (-4) & = & (-)(-)(12 \div 4) & = & +3 \\ (-12) \div (+4) & = & (-)(+)(12 \div 4) & = & -3 \end{array}$$

16. When simplifying a term containing integers, we combine the consecutive signs from left to right two at a time.

$$(+9) \times (-2) \div (-6) = (+)(-)(-)(9 \times 2 \div 6) = (-)(-)(18 \div 6) = +3$$

☺ EXERCISE

1. Multiply the following

$$\begin{array}{llll} \text{(a) } (-7)(-5) & \text{(c) } (-7)(+5) & \text{(e) } (-9)(+6) & \text{(g) } (+4)(+5)(-2) \\ \text{(b) } (+7)(-5) & \text{(d) } (+9)(-6) & \text{(f) } (+9)(+6) & \text{(h) } (-2)(-7)(+5) \end{array}$$

Answer: (a) +35 (b) -35 (c) -35 (d) -54 (e) -54 (f) +54 (g) -40 (h) +70

2. Divide the following

(a) $(-10) \div (-5)$ (c) $(-10) \div (+5)$ (e) $(-9) \div (+3)$ (g) $(+45) \div (-9)$
(b) $(+10) \div (-5)$ (d) $(+9) \div (-3)$ (f) $(+9) \div (+3)$ (h) $(-91) \div (+13)$

Answer: (a) +2 (b) -2 (c) -2 (d) -3 (e) -3 (f) +3 (g) -5 (h) -7

3. Reduce the following

(a) $(+3) \times (-4) \div (+6)$ (b) $(-15) \div (-5) \times (+4)$ (c) $(-21) \div (+14) \times (+2)$

Answer: (a) -2 (b) +12 (c) -3

☺ **L2 Lesson Plan 1: Check your Understanding**

1. What is the purpose of 0 on a number line containing integers?
2. How are integers defined by 0?
3. Arrange the following integers from the smallest to the largest.
+2, -3, +4, -5, +6, -7

Check your answers against the answers given below.

Answer:

- 1) The "0" always provides the reference point for the integers.
- 2) Positive integers are counted from 0 in the increasing direction. Negative integers are counted from 0 in the decreasing direction
- 3) -7, -5, -3, +2, +4, +6