

SOLUTIONS: Stage I Question Set 8

Solution to Question #1:

In order to get a "7", Barry would have to roll a "1". Since he has equal chances to roll a 1, 2, 3, 4, 5 or 6, his odds that his total score will be "7" are $1/6$.

The correct answer is (a).

Solution to Question #2:

$$(a - b) = 5 - 1000 = -995; ab = 5000$$

$$(a - b) + ab = -995 + 5000 = 4005$$

$$\text{Alternatively: } (a - b) + (ab) = 5 - 1000 + 5000 = 5 + 4000 = 4005$$

The correct answer is (d).

Solution to Question #3:

Currently, Henry pays $0.3(40,000) = \$12,000$ on income taxes.

If he moved to a jurisdiction where the rates were 10%, he would pay $0.1(40,000) = \$4,000$

$\$12,000 - \$4,000 = \$8,000$. Thus, Henry would save \$8,000 per year in income taxes by moving to the other jurisdiction.

$$\text{Alternatively: } \text{Henry would pay } (30 - 10)\% = 20\% = \$0.20(40,000) = \$8,000 \text{ less}$$

The correct answer is (d).

Solution to Question #4:

$$(1 + 2 + 4 + 5 + 2 + 4)/6 = 18/6 = 3.$$

The correct answer is (c).

Solution to Question #5:

Lynne has 23 minutes left, and $23 \times 60 = 1380$ seconds.

The correct answer is (b).

Solution to Question #6:

The small cubes have volume of $2 \times 2 \times 2 = 8$.

The large cube has a volume of $32 \times 32 \times 32 = 32768$

$$32768 \div 8 = 4096.$$

Alternatively: $32 / 2 = 16$ small cubes will define each side of the larger cube. Therefore, number of small cubes that will fit the larger cube = $16 \times 16 \times 16 = 4096$.

The correct answer is (c).

Solution to Question #7:

$1 \text{ m} = 39.37/12 \sim 3.28$ feet. Therefore, $1 \text{ m}^2 = (3.28)^2 \sim 10.76 \text{ sq. ft.}$

$10\text{m}^2 = 10(3.28)(3.28) \sim 107.6$ square feet ~ 108 square feet, to the nearest whole number.

The correct answer is (e).

Solution to Question #8:

a) 2 squares which are $15 \text{ cm} \times 15 \text{ cm}$

The total surface area of the square is $30 \text{ cm} \times 30 \text{ cm} = 900 \text{ cm}^2$

This cut would only take up $(2)(15 \times 15) = 450 \text{ cm}^2$

Only half of the square would be accounted for with this cut.

b) 4 squares which are $15 \text{ cm} \times 15 \text{ cm}$. This cut would account for the entire square as shown in the figure.

$$4(15 \times 15) = 4(225) = 900$$

c) 6 squares which are $5 \text{ cm} \times 5 \text{ cm}$

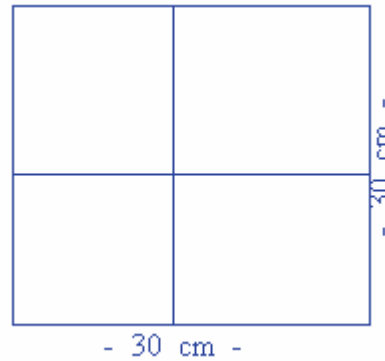
This cut would only take up $6(5 \times 5) = 150 \text{ cm}^2$ and would not account for the entire surface area.

- d) all of the above
False, since a) and c) are false.
- e) none of the above
False, since b) is correct.
- The correct answer is (b).

Solution to Question #9:

$$\frac{m - n}{m^2} = \frac{n + 1 - n}{(n + 1)^2} = \frac{1}{(n + 1)^2}$$

The correct answer is (c).



Solution to Question #10:

- a) If $n = -1$, then $mn = -25$ which is less than zero, and outside the interval given.
- b) If $n = 1$, then $mn = 25$ which is greater than 1, and outside the interval given.
- c) If $n = 0$, then $mn = 0$, which is outside the interval given, since $mn > 0$.
- d) If $n = 1/25$, then $mn = 1$, which is outside the interval given, since $mn < 1$.
- e) True, since none of a) - d) are possible values.

Alternatively: Dividing all elements by m we get, $0 < n < 1/25$

The correct answer is (e).

Solution to Question #11:

The shaded region = (the area of the square) - (the area of the quarter-circle).

The area of the square = $r^2 = 16\text{cm}^2$, since the radius = 4 cm.

The area of the quarter circle = $\pi (16)/4 = 4\pi$

The area of the shaded region = $16 - 4\pi$.

The correct answer is (d).

Solution to Question #12:

x cannot equal 300, since at that point, the equation is undefined.

If x is a negative number, the result of the equation will always be less than zero.

If $x < 300$, the result of the equation will always be negative.

If $300 < x < 500$, the result will always be positive.

If $x > 500$, the result will be negative.

The integer for which the result is a maximum is $x = 301$. If $x > 301$, the result keeps on shrinking as the absolute value of the denominator increases and the absolute value of the numerator decreases.

The correct answer is (a).

Solution to Question #13:

$x/y = 2x$, which is less than 0.

$y/x = 2/x$, which is less than 0.

$(x - y) = x - 1/2$, which is less than 0.

$y - x = 1/2 - x$, which is always greater than 0 for all the given values of x , since subtracting a negative number is the same as adding a positive number.

$0 = 0$

The largest element in the set is $(y - x)$, since that is the only positive number.

The correct answer is (d).

Solution to Question #14:

$4 \times 270 = 1080$ hours studying

The correct answer is (a).

Solution to Question #15:

The interior surface area of the cube would be $5(4 \times 4) = 80 \text{ cm}^2$ without the rectangular solid in it. The rectangular solid partially obscures three surfaces of the cube, and takes away an area of $[(1 \times 2) + (2 \times 3) + (1 \times 3)]$, but also adds

an area of $[(1 \times 2) + (2 \times 3) + (1 \times 3)]$ on the other surfaces, so there is no net gain or loss. The interior area of the figure is 80 cm^2 . The correct answer is **(d)**.

Solution to Question #16:

The outer square has a side of 16 cm, so the circle has a radius of 8 cm, and a diameter of 16 cm. The diagonal of the smaller square is equal to the diameter of the circle.

The diagonal = $s\sqrt{2}$, where s is the side of the small square. $s = 16/\sqrt{2} = 8\sqrt{2}$

The area of the small square = $(8\sqrt{2})^2 = 128 \text{ cm}^2$.

The area of the circle = $\pi (8)(8) = 64\pi$

The area of the circle - the area of the small square = the area of the shaded region

The area of the shaded region = $(64\pi - 128) \text{ cm}^2$

The correct answer is **(b)**.

Solution to Question #17:

What fraction represents the proportion of the area of triangle ADC to the area of triangle BCD?

The area of triangle ADC = $(1/2)(4)(10) = 20 \text{ m}^2$

The area of triangle BCD = $(1/2)(8)(10) = 40 \text{ m}^2$

The (area of triangle ADC)/(area triangle BCD) = $20/40 = 1/2$

The correct answer is **(b)**.

Solution to Question #18:

The first leap year in this interval was in 1904. The last leap year in this interval was in 1992.

$1904 \div 4 = 476$; $1992 \div 4 = 498$

Which means there are 22 intervals. Thus, there must be 23 leap years between 1901 and 1995.

The correct answer is **(c)**.

Solution to Question #19:

Valerie ate 2 pieces, since she ate $1/6$ of one of the pizzas.

The other nine members consumed $(2/3)(24) = 16$ pieces of pizza.

Altogether, 18 pieces of pizza were eaten, so 6 pieces were left over.

The correct answer is **(b)**.

Solution to Question #20:

For numbers between 50 and 100, you have the following:

54, 63, 72, 81, 90

For numbers between 100 and 200, you have the following:

108, 117, 126, 135, 144, 153, 162, 171, 180

Altogether, there are 14 numbers.

The correct answer is **(d)**.