ELEMENTS OF ALGEBRA.

I. DEFINITIONS.

NOTE TO TEACHERS.—Articles 1 to 15 may be omitted until the pupil reviews the book.

Article 1. In Algebra, quantities are represented by letters of the alphabet.

2. Quantity is any thing that is capable of increase or decrease; as, numbers, lines, space, time, etc.

3. Quantity is called magnitude, when considered in an undivided form; as, a quantity of water.

4. Quantity is called multitude, when made up of individual and distinct parts; as, three cents, a quantity composed of three single cents.

5. One of the single parts of which a quantity of multitude is composed, is called the unit of measure; thus, 1 cent is the unit of measure of the quantity 3 cents.

The value or measure of any quantity is the number of times it contains its unit of measure.

6. In quantities of magnitude, where there is no natural unit, it is necessary to fix upon an artificial unit as a standard of measure; then, to find the value of the quantity, we ascertain how many times it contains its unit of measure. Thus,

To measure the length of a line, take a certain assumed

Review.—1. How are quantities represented in Algebra? 2. What is quantity? 3. When called magnitude?
4. When multitude? 5. What is the unit of measure? 6. How find the value of a quantity when there is no natural unit?
distance called a foot, and, applying it a certain number
of times, say 5, it is found that the line is 5 feet long; in
this case, 1 foot is the unit of measure.

7. The Numerical Value of a quantity is the num-
ber that shows how many times it contains its unit of
measure.

Thus, the numerical value of a line 5 feet long, is 5.
The same quantity may have different numerical values,
according to the unit of measure assumed.

8. A Unit is a single thing of an order or kind.

9. Number is an expression denoting a unit, or a collection
of units. Numbers are either abstract or concrete.

10. An Abstract Number denotes how many times a
unit is to be taken.

A Concrete Number denotes the units that are taken.

Thus, 4 is an abstract number, denoting merely the num-
ber of units taken; while 4 feet is a concrete number, denot-
ing what unit is taken, as well as the number taken.

Or, a concrete number is the product of the unit of measure
by the corresponding abstract number. Thus, $6 \text{ equal } \$1$
multiplied by 6, or $\$1 \text{ taken 6 times}.$

11. In algebraic computations, letters are considered
the representatives of numbers.

12. There are two kinds of questions in Algebra, theo-
rems and problems.

13. In a Theorem, it is required to demonstrate some
relation or property of numbers, or abstract quantities.

14. In a Problem, it is required to find the value of
some unknown quantity, by means of certain given rela-
tions existing between it and others, which are known.

Review.—7. Define numerical value. 8. What is a unit? 9. A
number? 10. What does an abstract number denote? A concrete
number? 11. What do the letters used in Algebra represent?
12. How many kinds of questions in Algebra? 13. What is a the-
15. **Algebra** is a general method of solving problems and demonstrating theorems, by means of *figures, letters,* and *signs.* The letters and signs are called *symbols.*

**EXPLANATION OF SIGNS AND TERMS.**

16. **Known Quantities** are those whose values are given; **Unknown Quantities,** those whose values are to be determined.

17. Known quantities are generally represented by the first letters of the alphabet, as $a$, $b$, $c$, etc.; unknown quantities, by the last letters, as $x$, $y$, $z$.

18. The principal signs used in Algebra are

$$=, +, -, \times, \div, ( ), >, v.'$$

Each sign is the representative of certain words. They are used to express the various operations in the clearest and briefest manner.

19. The **Sign of Equality,** $=$, is read *equal to.* It denotes that the quantities between which it is placed are equal. Thus, $a=3$, denotes that the quantity represented by $a$ is equal to 3.

20. The **Sign of Addition,** $+$, is read *plus.* It denotes that the quantity to which it is prefixed is to be added.

Thus, $a+b$, denotes that $b$ is to be added to $a$. If $a=2$ and $b=3$, then $a+b=2+3$, which $=5$.

21. The **Sign of Subtraction,** $-$, is read *minus.* It denotes that the quantity to which it is prefixed is to be subtracted.

Thus, $a-b$, denotes that $b$ is to be subtracted from $a$. If $a=5$ and $b=3$, then $5-3=2$.

22. The signs $+$ and $-$ are called the signs. The former is called the \textit{positive}, the latter the \textit{negative} sign: they are said to be \textit{contrary} or \textit{opposite}.

23. Every quantity is supposed to be preceded by one of these signs. Quantities having the positive sign are called \textit{positive}; those having the negative sign, \textit{negative}.

When a quantity has no sign prefixed, it is positive.

24. Quantities having the same sign are said to have \textit{like} signs; those having different signs, \textit{unlike} signs.

Thus, $+a$ and $+b$, or $-a$ and $-b$, have like signs; while $-c$ and $-d$ have unlike signs.

25. The \textbf{Sign of Multiplication}, $\times$, is read \textit{into}, or \textit{multiplied by}. It denotes that the quantities between which it is placed are to be multiplied together.

The product of two or more letters is sometimes expressed by a dot or point, but more frequently by writing them in close succession without any sign. Thus, $ab$ expresses the same as $a \times b$ or $a \cdot b$, and $abc = a \times b \times c$, or $a \cdot b \cdot c$.

26. \textbf{Factors} are quantities that are to be multiplied together.

The \textit{continued product} of several factors means the product of the first and second multiplied by the third, this product by the fourth, and so on.

Thus, the continued product of $a$, $b$, and $c$, is $a \times b \times c$, or $abc$. If $a = 2$, $b = 3$, and $c = 5$, then $abc = 2 \times 3 \times 5 = 6 \times 5 = 30$.

27. The \textbf{Sign of Division}, $\div$, is read \textit{divided by}. It

\textbf{Review}.—22. What are the signs plus and minus called, by way of distinction? Which is positive? Which negative?

23. What are quantities preceded by the sign plus said to be? By the sign minus? When no sign is prefixed? 24. When do quantities have like signs? When unlike signs?

25. How is the sign $\times$ read, and what does it denote? What other methods of representing multiplication? 26. What are factors? How many in $a$? In $ab$? In $abc$? In $5abc$?

27. How is the sign $\div$ read, and what does it denote? What other methods of representing division?
denotes that the quantity preceding it is to be divided by
that following it. Division is oftener represented by
placing the dividend as the numerator, and the divisor as the
denominator of a fraction.

Thus, $a \div b$, or $\frac{a}{b}$, means, that $a$ is to be divided by $b$.
If $a=12$ and $b=3$, then $a \div b = 12 \div 3 = 4$; or $\frac{12}{3} = 4$.

Division is also represented thus, $a \mid b$, or $b \mid a$, $a$ denoting
the dividend, and $b$ the divisor.

28. The Sign of Inequality, $>$, denotes that one of
the two quantities between which it is placed is greater
than the other. The opening of the sign is toward the
greater quantity.

Thus, $a > b$, denotes that $a$ is greater than $b$. It is read,
a greater than $b$. If $a=5$, and $b=3$, then $5 > 3$. Also,
$c < d$, denotes that $c$ is less than $d$. It is read, $c$ less than $d$.
If $c=4$ and $d=7$, then $4 < 7$.

29. The Sign of Infinity, $\infty$, denotes a quantity greater
than any that can be assigned, or one indefinitely great.

30. The Numeral Coefficient of a quantity is a num-
ber prefixed to it, showing how many times the quantity
is taken.

Thus, $a + a + a + a = 4a$ ; and $ax + ax + ax = 3ax$.

31. The Literal Coefficient of a quantity is a quantity
by which it is multiplied. Thus, in the quantity $ax$, $a$ may
be considered the coefficient of $y$, or $y$ the coefficient of $a$.
The literal coefficient is generally regarded as a known
quantity.

32. The coefficient of a quantity may consist of a num-
ber and a literal part. Thus, in $5ax$, $5a$ may be re-

Review.—28. What is the sign $>$ called, and what does it de-
ote? Which quantity is placed at the opening?
29. What does the sign $\infty$ denote? 30. What is a numeral co-
efficient? How often is $ax$ taken in $3ax$? In $5ax$? In $7ax$?
31. What is a literal coefficient? 32. When a quantity has no
coefficient, what is understood?
garded as the coefficient of \( x \). If \( a = 2 \), then \( 5a = 10 \), and \( 5ax = 10x \).

When no numeral coefficient is prefixed to a quantity, its coefficient is understood to be unity. Thus, \( a = 1a \), and \( bx = 1bx \).

33. The Power of a quantity is the product arising from multiplying the quantity by itself one or more times.

When the quantity is taken twice as a factor, the product is called its square, or second power; when three times, the cube, or third power; when four times, the fourth power, and so on.

Thus, \( a \times a = aa \), is the second power of \( a \); \( a \times a \times a = aaa \), is the third power of \( a \); \( a \times a \times a \times a = aaaa \), is the fourth power of \( a \).

An Exponent is a figure placed at the right, and a little above a quantity, to show how many times it is taken as a factor.

Thus, \( aa = a^2 \); \( aaa = a^3 \); \( aaaa = a^4 \); \( aabbb = a^2b^2 \).

When no exponent is expressed, it is understood to be unity. Thus, \( a \) is the same as \( a^1 \), each expressing the first power of \( a \).

34. To raise a quantity to any given power is to find that power of the quantity.

35. The Root of a quantity is another quantity, some power of which equals the given quantity. The root is called the square root, cube root, fourth root, etc., according to the number of times it is taken as a factor to produce the given quantity.

Thus, \( a \) is the second or square root of \( a^2 \), since \( a \times a = a^2 \). So, \( x \) is the third or cube root of \( x^3 \), since \( x \times x \times x = x^3 \).

36. To extract any root of a quantity is to find that root.

Review.—33. What is the power of a quantity?  What is the second power of \( a \)?  The third power of \( a \)?

33. What is an exponent?  For what used?  How many times is \( x \) taken as a factor in \( x^2 \)?  In \( x^3 \)?  In \( x^4 \)?  Where no exponent is written, what is understood?  35. What is the root of a quantity?
37. The Radical Sign, √, placed before a quantity, indicates that its root is to be extracted.

Thus, √a, or \(\sqrt{a}\), denotes the square root of \(a\); \(\sqrt[3]{a}\) denotes the cube root of \(a\); \(\sqrt[4]{a}\) denotes the fourth root of \(a\).

38. The number placed over the radical sign is called the index of the root. Thus, 2 is the index of the square root, 3 of the cube root, 4 of the fourth root, and so on. When the radical has no index over it, 2 is understood.

39. Every quantity or combination of quantities expressed by means of symbols, is called an algebraic expression.

Thus, \(3a\) is the algebraic expression for 3 times the quantity \(a\); \(3a - 4b\), for 3 times \(a\), diminished by 4 times \(b\); \(2a^2 + 3ab\), for twice the square of \(a\), increased by 3 times the product of \(a\) and \(b\).

40. A Monomial, or Term, is an algebraic expression, not united to any other by the sign + or —.

A monomial is sometimes called a simple quantity. Thus, \(a\), \(3a\), \(-a^2b\), \(2any^2\), are monomials, or simple quantities.

41. A Polynomial is an algebraic expression, composed of two or more terms.

Thus, \(c - 2d - b\) is a polynomial.

42. A Binomial is a polynomial composed of two terms. Thus, \(a - b\), \(-a - b\), and \(c - d\), are binomials.

A Residual Quantity is a binomial, in which the second term is negative, as \(a - b\).

43. A Trinomial is a polynomial consisting of three terms. Thus, \(a + b + c\), and \(a - b - c\), are trinomials.

44. The Numerical Value of an algebraic expression

is the number obtained, by giving particular values to the letters, and then performing the operations indicated.

In the algebraic expression $2a + 3b$, if $a = 4$, and $b = 5$, then $2a = 8$, and $3b = 15$, and the numerical value is $8 + 15 = 23$.

45. The value of a polynomial is not affected by changing the order of the terms, provided each term retains its respective sign. Thus, $a^2 + 2a + b = b + a^2 + 2a$. This is self-evident.

46. Each of the literal factors of any simple quantity or term is called a *dimension* of that term. The *degree* of a term depends on the number of its literal factors.

Thus, $ax$ consists of two literal factors, $a$ and $x$, and is of the *second degree*. The quantity $a^2b$ contains three literal factors, $a$, $a$, and $b$, and is of the *third degree*. $2a^2x^3$ contains 5 literal factors, $a$, $a$, $a$, $x$, and $x$, and is of the *fifth degree*; and so on.

47. A polynomial is said to be *homogeneous*, when each of its terms is of the same degree.

Thus, the polynomials $2a - 3b + c$, of the first degree, $a^2 + 3bc + xy$, of the second degree, and $x^3 - 8ay^2$, of the third degree, are homogeneous: $a^3 + x^2$ is not homogeneous.

48. A *Parenthesis*, ( ), is used to show that all the included terms are to be considered together as a single term.

Thus, $4(a - b)$ means that $a - b$ is to be multiplied by $4$; $(a + x)(a - x)$ means that $a + x$ is to be multiplied by $a - x$; $10 - (a + c)$ means that $a + c$ is to be subtracted from $10$; $(a - b)^2$ means that $a - b$ is to be raised to the second power; and so on.

49. A *Vinculum*, ———, is sometimes used instead of

Review. — 46. What is the dimension of a term? On what does the degree of a term depend? What is the degree of the term $xy$? Of $xyz$? Of $2axy$? Of $x^2$? 47. When is a polynomial homogeneous? 48. For what is a parenthesis used? 49. What is a vinculum, and for what used?
DEFINITIONS AND NOTATION.

a parenthesis. Thus, \( \overline{a-b} \times x \) means the same as \((a-b)x\). Sometimes the vinculum is placed vertically: it is then called a bar.

Thus, \( a_y^2 \) has the same meaning as \((a-x+4)y^2\).

\[
\begin{array}{c}
-x \\
+4 \\
\end{array}
\]

50. Similar or Like quantities are those composed of the same letters, affected with the same exponents.

Thus, \( 7ab \) and \( -3ab \), also \( 4a^2b^3 \) and \( 7a^3b^4 \), are similar terms; but \( 2a^2b \) and \( 2ab^3 \) are not similar; for, though composed of the same letters, these letters have different exponents.

51. The Reciprocal of a quantity is unity divided by that quantity. Thus, the reciprocal of \( 2 \) is \( \frac{1}{2} \), of \( a \) is \( \frac{1}{a} \).

The reciprocal of \( \frac{3}{2} \) is \( \frac{3}{2} \), or \( \frac{2}{3} \). Hence, the reciprocal of a fraction is the fraction inverted.

52. The same letter accented is often used to denote quantities which occupy similar positions in different equations or investigations.

Thus, \( a, a', a'', a''' \), represent four different quantities; read \( a, \) a prime, \( a \) second, \( a \) third, and so on.

EXAMPLES.

The following examples are intended to exercise the learner in the use and meaning of the signs.

Copy each example on the slate or blackboard, and then express it in common language.

Let the numerical values be found, on the supposition that \( a=4, b=3, c=5, d=10, x=2, \) and \( y=6 \).

1. \( c+d-b \) . . . Ans. 12.  5. \( ay + \frac{cd}{b} \) . . . . Ans. 33.
2. \( 4a-x \) . . . Ans. 14.  6. \( 3a^2+2cx-b^3 \) . Ans. 41.
3. \( -3ax \). . . Ans. -24.  7. \( a(a+b) \) . . . . Ans. 28.
4. \( 6a^2x \) . . . Ans. 192.

Review.—50. What are similar or like quantities? 51. The reciprocal of a quantity? 52. What the use of accented letters?
8. \(a+b \times a-b\) .............................. Ans. 13.
9. \((a+b)(a-b)\) ................................. Ans. 7.
10. \(x^3-3(a+x)(a-x)+2by\) ................. Ans. 4.
11. \(\frac{2ax^3}{(a-x)^2} - 6xy/a\) ................. Ans. -16.
12. \(3(a+c)(c-a)+3c^2-3a^3\) ................ Ans. 54.
13. \(\frac{a^3-x^3}{a-x}\) ............................. Ans. 4.

In the following, convert the words into algebraic symbols:
1. Three times \(a\), plus \(b\), minus four times \(c\).
2. Five times \(a\), divided by three times \(b\).
3. \(a\) minus \(b\), into three times \(c\).
4. \(a\), minus three times \(b\) into \(c\).
5. \(a\) plus \(b\), divided by three \(c\).
6. \(a\), plus \(b\) divided by three \(c\).
7. \(a\) squared, minus three \(a\) into \(b\), plus 5 times \(c\) into \(d\) squared.
8. \(x\) cubed minus \(b\) cubed, divided by \(x\) squared minus \(b\) squared.
9. Five \(a\) squared, into \(a\) plus \(b\), into \(c\) minus \(d\), minus three times \(x\) fourth power.
10. \(a\) squared plus \(b\) squared, divided by \(a\) plus \(b\), squared.
11. The square root of \(a\), minus the square root of \(x\).
12. The square root of \(a\) minus \(x\).

**Answers.**

1. \(3a+b-4c\)
2. \(\frac{5a}{3b}\)
3. \((a-b)3c\)
4. \(a-3bc\)
5. \(\frac{a+b}{3c}\)
6. \(a+\frac{b}{3c}\)
7. \(a^2-3ab+5cd^2\)
8. \(x^3-b^3\)
9. \(5a^2(a+b)(c-d)-3x^4\)
10. \(\frac{a^2+b^2}{(a+b)^2}\)
11. \(\sqrt{a}-\sqrt{x}\)
12. \(\sqrt{a-x}\)