Review of Equations

The purpose of this appendix is to refresh your memory about equations and how they are solved.

We start with linear equations (they are called linear because you get a straight line when you plot the equation on a graph) with only one variable: x. You will also encounter equations with denominators. These can be proportions, which consist of two equal fractions, such as x over 4 equals 2 over 3, or non-proportions, when there are additional terms in the equation.

The next step is equations with two variables: x and y. These are called simultaneous equations, and we solve them to find out the numbers that x and y represent.

Finally, you learn to solve quadratic equations; they are called quadratic because they involve a square term, such as x^2. They are solved either by calculating a square root, if there are only squared terms in the equation, or by factoring.

Linear Equations With One Variable

When we solve equations, we want to find out what **number** x stands for. In other words, we want a new equation that states x = a number.

Example:

Solve: $x + 5 = 7$.

Subtract 5 from both sides: $x = 2$

Check: $2 + 5 = 7$

Example:

Solve: $x - 5 = 7$.

Add 5 to both sides: $x = 12$

Check: $12 - 5 = 7$

Example:

Solve: $3x = 18$. *Note:* $3x$ means 3 times x.

Divide both sides by 3: $x = 6$

Check: $3(6) = 18$

Example:

Solve: $\frac{x}{3} = 18$. *Note*: This means x divided by 3.

Multiply both sides by 3: $3\left(\frac{x}{3}\right) = 3(18)$

$x = 54$

Check: $\frac{54}{3} = 18$

Example:

Solve: $22 = x + 15$. Here x is on the right side of the equation but that does not matter.

Subtract 15 from both sides.

$7 = x$ or $x = 7$

Check: $7 + 15 = 22$

Example:

Solve: $2x - 11 = 7$

Here we have two operations: multiplication and subtraction. The order of operations in solving equations is opposite to that in arithmetic. We "undo" what has been done before.

First add 11 to both sides: $2x = 18$

Divide by 2: $x = 9$

Check: $2(9) - 11 = 18 - 11 = 7$

Example:

Solve: $\frac{x}{3} + 10 = 14$

Subtract 10 from both sides: $\frac{x}{3} = 4$

Multiply both sides by 3: $3\left(\frac{x}{3}\right) = 3(4)$

$x = 12$

Check: $\frac{12}{3} + 10 = 4 + 10 = 14$

Example:

Solve: $\frac{2x}{3} + 5 = 15$

Subtract 5 from both sides: $\frac{2x}{3} = 10$

Multiply by 3: $3\left(\frac{2x}{3}\right) = 3(10)$

$2x = 30$

$x = 15$

Check: $\frac{2(15)}{3} + 5 = 10 + 5 = 15$

Example:

$40 - 7x = 60 - 5x$

Should we add $7x$ or 5x? It is easier if we add $7x$ to both sides, because then we avoid a minus sign before the x-term.

Add $7x$ to both sides: $40 = 60 + 2x$

Subtract 60 from both sides: $-20 = 2x$

Divide by 2: $-10 = x$

Check: Left side $= 40 - 7(-10) = 40 + 70 = 110$ Multiplying two negatives $\rightarrow$ positive.

Right side: $60 - 5(-10) = 60 + 50 = 110$

Example:

Solve: $5 + (3x - 4) = 7 - (2x - 9)$

Remove the parentheses: $5 + 3x - 4 = 7 - 2x + 9$

Simplify both sides: $1 + 3x = 16 - 2x$

Add $2x$ to both sides: $1 + 5x = 16$

Subtract 1 from both sides: $5x = 15$

Divide by 5: $x = 3$

Check: Left side $= 5 + (3(3) - 4) = 5 + 5 = 10$

Right side $= 7 - (2(3) - 9) = 7 - (6 - 9)$

$= 7 - (-3) = 7 + 3 = 10$

Equations With Denominators

Proportions

Example:

Solve: $\frac{x}{4} = \frac{3}{2}$

An equation that consists of two equal fractions only is called a proportion. It is usually solved by cross multiplication:

$2x = 4(3)$

$2x = 12$

$x = 6$

Check: $\frac{6}{4} = \frac{3}{2}$

Example:

Solve $\frac{5}{x} = \frac{10}{4}$

Cross multiply: $5(4) = 10x$

$20 = 10x$

$x = 2$

Check: $\frac{5}{2} = 2.5$ $\frac{10}{4} = 2.5$

Example:

Solve: $\frac{2x+3}{12} = \frac{x}{3}$

Cross multiply: $3(2x+3) = 12x$

$6x + 9 = 12x$

$9 = 6x$

$x = \frac{3}{2}$ or $x = 1.5$

Check:

$$\frac{2\left(\frac{3}{2}\right)+3}{12}=\frac{3+3}{12}=\frac{6}{12}=\frac{1}{2}$$

$$\frac{\frac{3}{2}}{3}=\frac{3}{2(3)}=\frac{1}{2}$$

Non-Proportion Equations

Example:

Solve: $\frac{x}{2}=9-\frac{x}{4}$

This equation is not a proportion because it contains three terms. A proportion consists of two equal fractions only.

To solve this equation we must find the least common denominator (LCD), which in this case is 4. Then we multiply all terms by 4.

$$4\left(\frac{x}{2}\right)=4(9)-4\left(\frac{x}{4}\right)$$

$$2x=36-x$$

Add x: $3x=36$

$$x=12$$

Check: $\frac{12}{2}=6 \qquad 9-\frac{12}{4}=9-3=6$

Example:

Solve: $\frac{x}{3}-\frac{x}{5}=2$ LCD is 15.

$$15\left(\frac{x}{3}\right)-15\left(\frac{x}{5}\right)=15(2)$$
$$5x-3x=30$$
$$2x=30$$
$$x=15$$

Check: $\frac{15}{3}-\frac{15}{5}=5-3=2$

Simultaneous Equations

When solving word problems, we often need two variables: x and y. To solve for both, we need two equations.

Example:

Solve for x and y:

$$x+y=6$$
$$x-y=2$$

Add the equations:

$$2x=8$$
$$x=4$$
$$4+y=6$$
$$y=2$$

Check: $4+2=6$ $\quad 4-2=2$

Example:

Solve for x and y:

$5x-2y=5$ Multiply by 3. $\quad 15x-6y=15$

$x+3y=18$ Multiply by 2. $\quad 2x+6y=36$

Add: $\quad 17x=51$

$$x=3$$

Replace x in one of the original equations:

$$3+3y=18$$
$$3y=15$$
$$y=5$$

Check: $5(3)-2(5)=15-10=5$

$3+3(5)=3+15=18$

Alternate solution:

Multiply the second equation by 5: $5x+15y=90$

Subtract the first equation from the second: $5x-2y=5$

$$17y=85$$
$$y=5$$
$$5x-2(5)=5$$
$$5x=15$$
$$x=3$$

Example:

Solve for x and y:

$$y=x+3$$
$$2x+y=9$$

Here we could rewrite the first equation as $-x+y=3$ but it is easier to substitute the first equation into the second:

$$2x+x+3=9$$
$$3x=6$$
$$x=2$$
$$y=2+3=5$$

Check: $2(2)+5=4+5=9$

Quadratic Equations

Sometimes our equations contain an x^2-term. If there is no x-term, we can solve the equation by taking the square root of both sides.

Example:

Solve $x^2 = 25$

$$x = \pm\sqrt{25}$$

$$x = \pm 5$$

The x-term can be 5 or –5. However, in a word problem we must be sure that we can use the negative answer. For example, if the problem stated: The square of a whole number is 25, we have to reject –5, because the whole numbers are 0, 1, 2, 3,.... but if the problem stated: The square of an integer is 25, we give both 5 and –5 as answers.

Often quadratic equations also contain an x-term. Such equations have to be solved by factoring or formula. We are not going to cover the formula here.

In order to factor, we need all terms on one side of the equation. It is usually easiest to have them to the left. Then we look at the constant term, in the following example +5. We need two integers whose product is +5. Here we have to guess –1 and –5. Then we add the two numbers we found. Do they add up to the coefficient of x (that is, the number before x)? In this case: $-1 + -5 = -6$.

Example:

Solve: $x^2 - 6x + 5 = 0$.

Factor: $(x - 5)(x - 1) = 0$

This is a true statement, if each factor equals zero.

We set each factor = 0.

$$x - 5 = 0 \qquad x - 1 = 0$$
$$x = 5 \qquad x = 1$$

Answer: x can either be 5 or 1.

Example

Solve: $x^2 - 6x + 9 = 0$

Factor: $(x - 3)(x - 3) = 0$

$$x - 3 = 0 \qquad x - 3 = 0$$
$$x = 3 \qquad x = 3$$

The two solutions are both equal to 3.

Example:

Solve $x^2 + x - 2 = 0$

Factor: $(x - 1)(x + 2) = 0$.

$$x - 1 = 0 \qquad x + 2 = 0$$
$$x = 1 \qquad x = -2$$

Check: If $x = 1$ then $1^2 + 1 - 2 = 0$

If $x = -2$ then $(-2)^2 + (-2) - 2 = 4 - 2 - 2 = 0$

Answer: $x = 1$ or $= -2$.