Chapter 1

Simple Equation Problems

In order to solve mathematical word problems we often need to use equations. In this chapter, you will learn how to set up simple equations to solve different kinds of word problems. For example, we will cut up a length of board or rope into shorter and longer pieces and, given the known total length and other facts, we will calculate the lengths of the pieces cut from it. In other examples we will calculate the ages of two children once we know how many years they are apart and what the sum of their ages is. We will also look at situations where one person weighs more or less than another and calculate each person's weight from the information given in the problem.

Then, we will learn the mathematical symbols for inequalities, that is, situations where something is greater than or smaller than something else and also how to solve problems in which we are told that something is at most so big or that something costs at least so much.

Finally, we will tackle word problems involving all kinds of numbers: positive and negative integers, including zero; odd and even integers; and consecutive integers.

The last example will show you how to solve a problem that requires the use of a quadratic equation.

(Note: If you want to brush up on your skills for solving equations, see the Appendix.)

Length Problems

Example:

Cut a 10-foot (ft.) long piece of wood into two pieces so that one piece is 2 ft. longer than the other. To solve this problem you have two choices:

By using algebra:

Call one piece x, then the other piece is $x + 2$.

Write an equation: $x + x + 2 = 10$

Solve the equation:

$$2x = 8$$
$$x = 4$$
$$x + 2 = 6$$
$$\text{Total} = 10$$

Or by using arithmetic:

$10 - 2 = 8$ Take away the 2 ft. from the whole piece.

$8 \div 2 = 4$ Divide the piece by 2.

$4 + 2 = 6$ Add the 2 ft. to one of the pieces.

Total = 10

The pieces were 4 ft. and 6 ft.

Check your work by adding the pieces. Together they were 10 ft. Read the problem again to check all the facts.

Example:

A length of board was 10 inches shorter than another length. Together the boards were 20 inches. How long were the boards?

Call the long piece x and the short piece $x - 10$.

$$x + x - 10 = 20$$
$$2x = 30$$
$$x = 15$$
$$15 - 10 = 5$$

The pieces were 5 and 15 in.

Reread the problem. Is it true that the pieces equal 20 in. when put together? Is one piece 10 in. shorter than the other?

Practice Problems:

1.1 Solve the previous problem by calling the short piece of board x.

1.2 A 12-ft. rope is cut into three pieces so that the second piece is 1 ft. longer than the first and the third piece is 1 ft. longer than the second. How long are the pieces?

1.3 A 9-ft. board is cut into two pieces so that one piece is twice the other. How long are the pieces?

1.4 An 80-in. board is to be cut into three pieces so that one piece is twice another and the third piece is 10 in. more than the second. Find the length of each piece.

1.5 Two ropes are together 275 yards long. One rope is 50% longer than the other. How long are the ropes?

Age Problems

Example:

Leah is 2 years older than Tracy. Together the girls are 10 years old. How old are they?

Call Leah's age x. Then Tracy's age is $x - 2$.

$$x + x - 2 = 10$$
$$2x = 12$$
$$x = 6$$
$$x - 2 = 4$$

Leah is 6 years old and Tracy is 4.

Do you recognize this problem as essentially the same as the first length problem?

Practice Problems:

1.6 Elsa is 7 years younger than Thor. The sum of their ages is 35. How old are they?

1.7 Kristina's grandmother is 12 times as old as Kristina. Together they are 91 years. How old is the grandmother?

1.8 The sum of the ages of Jessica, her mother, and her grandmother is 100 years. The grandma is twice as old as the mother. Jessica's mother is 28 years older than Jessica. How old is grandma?

1.9 The sum of Eric's age and Lucas's age is 65. Two times Eric's age is the same as three times Lucas's age. Find the ages of the men.

1.10 Michael's age is multiplied by 7. Then 9 is added. The result is 93. How old is Michael?

Use of the Words "More Than" and "Less Than"

Beware of keywords! Many people believe that if they see "more than" in a word problem they must add and with "less than" they must subtract. That is not *always* the case. Look at the following examples:

Examples:

a) Sue weighs 5 pounds more than Amanda. If Amanda weighs 103 pounds, how much does Sue weigh?

b) Sue weighs 5 pounds less than Amanda. If Sue weighs 103 pounds, how much does Amanda weigh?

c) A school has an enrollment of 2381 students. This is 53 students less than last year. What was the enrollment last year?

d) A school had an enrollment of 2381 students last year. This is 53 students less than this year. What is the enrollment this year?

In each of these examples should you add or subtract?

Solutions:

a) Who weighs more? Sue. So we *add* 5 pounds to Amanda's 103 pounds.

b) Who weighs more? Amanda. So we *add* 5 pounds to Sue's 103 pounds

c) Is the enrollment larger this year? No, it is smaller by 53 students. We *add* 53 students to 2381.

d) Last year's enrollment was smaller than this year's. We *add* 53 to 2381.

Answers:

a) and *b*) 108 pounds; *c*) and *d*) 2434.

Practice Problems:

1.11 Six less than a number is 20. Find the number.

1.12 The sun rose at 7:45 a.m. That is 3 minutes earlier than yesterday. At what time did the sun rise yesterday?

1.13 The water level in the reservoir was 67.3 ft. That is 0.1 ft. higher than a month ago. What was the water level a month ago?

1.14 The population of the village is 487. That is 16 people less than 5 years ago. What was the population 5 years ago?

1.15 Joan weights 143 pounds. That is 10 pounds more than she weighed 5 years ago. How much did she weigh 5 years ago?

Inequalities Using the Words "At Least" and "At Most"

The symbols below translate into words the following way:

$<$ is less than.

$\leq$ is less than or equal to.

$>$ is greater than.

$\geq$ is equal to or greater than.

In word problems these symbols can also be used to translate:

$\leq$ at most.

$\geq$ at least.

Example:

Translate and solve:
1/3 of an unknown number is less than 4.

$$x/3 < 4$$

Multiply both sides by 3: $x < 12$

The unknown number can be any number smaller than 12.

Example:

If a number is subtracted from 10, the result is at most 22.

$$10 - x \leq 22$$

Subtract 10: $-x \leq 12$

Multiply by –1: $x \geq -12$

Note: When we multiply or divide an inequality by a negative number, we must reverse the inequality sign!

Answer: x can be any number that is equal to or greater than –12.

Practice Problems:

1.16 Compare –6 and –10 by using inequality symbols.

Solve the following inequalities:

1.17 The difference of a number and 7 is greater than or equal to 11.

1.18 Two times a number subtracted from 19 is less than or equal to 3.

Number Problems

Number problems deal with numbers, usually integers (whole numbers, which includes zero, and their negatives). An equation can be written once the problem has been stated. If the problem states something special about a number, that number is usually called x. Often these numbers deal with consecutive integers, which are integers that follow in order such as 5, 6, 7,... or –4, –3, –2,... Consecutive odd or even integers are two numbers apart, such as 2, 4, 6,... or 1, 3, 5,... If you add $1 + 3 + 5 = 9$, we have an odd sum of odd numbers, but $1 + 3 + 5 + 7 = 16$. A sum of four odd integers gives an even sum. Can a sum of even consecutive integers ever be odd?

Example:

The sum of three consecutive integers is 6. Find the integers.

Call the smallest integer x. Then the others are $x + 1$ and $x + 2$

The sum is $x + x + 1 + x + 2$ or 6.

$$3x + 3 = 6$$
$$3x = 3$$
$$x = 1$$
$$x + 1 = 2$$
$$x + 2 = 3$$

The consecutive integers are 1, 2, and 3.

Check: $1 + 2 + 3 = 6$

Example:

The sum of four consecutive even integers is 44. Find the integers.

Call the integers x, $x + 2$, $x + 4$, $x + 6$.

The sum is $x + x + 2 + x + 4 + x + 6$ or 44.

$$4x + 12 = 44$$
$$4x = 32$$
$$x = 8$$
$$x + 2 = 10$$
$$x + 4 = 12$$
$$x + 6 = 14$$

The four integers are 8, 10, 12, and 14.

Check: $8 + 10 + 12 + 14 = 44$

Practice Problems:

1.19 Find five consecutive odd integers whose sum is 55.

1.20 The sum of three consecutive even integers is –12. Find the integers.

1.21 The sum of five consecutive integers is 0. Find the integers.

1.22 The sum of four consecutive even integers is 2 more than five times the first integer. Find the first integer.

1.23 Two consecutive even integers have a sum of 14. What is their product?

Some types of number problems may sound more complicated, but they can also be solved by simple equations.

Example:

Find three consecutive odd integers so that four times the first integer equals the sum of the other two.

Call the smallest integer x. The others are $x + 2$ and $x + 4$.

$$4x = x + 2 + x + 4$$

$$4x = 2x + 6$$

$$2x = 6$$

$$x = 3$$

$$x + 2 = 5$$

$$x + 4 = 7$$

The odd integers are: 3, 5, and 7.

Check: $4(3) = 12$ $\qquad$ $5 + 7 = 12$

Practice Problems:

1.24 Find the largest of three consecutive even integers when six times the first integer is equal to five times the middle integer.

1.25 The sum of four consecutive integers is 186. Find the integers.

1.26 Find three consecutive even integers such that three times the first equals the sum of the other two.

1.27 Five times an odd integer plus three times the next odd integer equals 62. Find the first odd integer.

1.28 Find three even integers, if five times the first integer, plus twice the second integer, plus the third integer equals 40.

The following example and practice problems require the knowledge of how to solve quadratic equations. If you have never done this, you might want to skip them. However, they are also explained in the Appendix. All of the following problems can be solved by factoring.

Example:

The product of two consecutive integers is 20. Find the integers.

Call the integers x and $x + 1$.

The product is $x(x + 1) = 20$

$$x^2 + x = 20$$

$$x^2 + x - 20 = 0$$

We factor this equation by finding two integers whose product is -20 and whose sum is 1 (the coefficient of x). The integers are -4 and 5.

$(x-4)(x+5)=0 \qquad x-4=0 \qquad x+5=0$

$x=4 \qquad x=-5$

$x+1=5 \qquad x+1=-4$

The consecutive integers are 4 and 5, or –5 and –4.

Note: Quadratic equations always have two possible solutions.

Practice Problems:

1.29 The product of two consecutive positive integers is 20. Find the integers.

1.30 One number is 4 more than another number. Their product is 5. Find the numbers.

1.31 The product of two consecutive odd integers is 15. Find the integers.

1.32 The product of two numbers is 243. One number is three times the other one. Find the numbers.

1.33 One number is 2 more than another. Their product is 440. Find the numbers.