MATH MILESTONE # B2

INTEGERS

The word, **milestone**, means "a point at which a significant change occurs." A Math Milestone refers to a significant point in the understanding of mathematics.

To reach this milestone one should know how to use integers to simplify math computations.

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Please consult the Glossary supplied with this Milestone for mathematical terms. Consult a regular dictionary at www.dictionary.com for general English words that one does not understand fully.

You may start with the Diagnostic Test on the next page to assess your proficiency on this milestone. Then continue with the lessons with special attention to those, which address the weak areas.

Researched and written by Vinay Agarwala
Edited by Ivan Doskocil

DIAGNOSTIC TEST

1. Convert the following operations with natural numbers to integers.
   (a) 0 – 7 → ___   (b) 0 + 5 → ___   (c) 0 – 35 → ___   (d) 0 + 56 → ___
2. Convert the following integers to operations with natural numbers.
   (a) −9 → ___   (b) +8 → ___   (c) −23 → ___   (d) +47 → ___
3. Draw a number line and show where the following numbers would appear:
   +15, 0, −15, −11, +3, −7, 8, +9, 20,
4. Insert the correct symbol (>, =, or <)
   (a) −7 __ +5   (b) +2 __ −5   (c) −3 __ −8   (d) +8 __ +11   (e) −9 __ +9
5. What number will you reach by counting on a number line
   (a) +7 from +5   (b) +8 from −3   (c) −7 from +5   (d) −8 from −3
6. Count the following integers together.
   (a) +2, −8 and +7   (b) −9, +3, +5 and −7   (c) +6, −5, −7, −12 and +3
7. Add the following
   (a) (−7) + (−5)   (b) (−7) + (+5)   (c) (−9) + (+6)   (d) (+4) + (+9)
8. Subtract the following
   (a) (−7) − (−5)   (b) (−7) − (+5)   (c) (+9) − (−6)   (d) (+4) − (+9)
9. Multiply the following.
   (a) (−7)(−5)   (c) (+6)(+3)   (e) (+3)(−4)(+5)   (g) (−8)(−2)(−4)
   (b) (−4)(+6)   (d) (+8)(−5)   (f) (−3)(+4)(−5)   (h) (+8)(+2)(+4)
10. Divide the following.
   (a) (−35) / (−5)   (c) (+39) / (+3)   (e) (+63) / (−9)   (g) (−56) / (−7)
   (b) (−48) / (+6)   (d) (+42) / (−7)   (f) (−72) / (+4)   (h) (+36) / (+9)

Answer: **1.** (a) −7 (b) +5 (c) −35 (d) +56 **2.** (a) 0 − 9 (b) 0 + 8 (c) 0 − 23 (d) 0 + 47 **4.** (a) < (b) > (c) > (d) < (e) < **5.** (a) +12 (b) +5 (c) −2 (d) −11 **6.** (a) +1 (b) −8 (c) −15 **7.** (a) −12 (b) −2 (c) −3 (d) +13 **8.** (a) −2 (b) −12 (c) +15 (d) −5 **9.** (a) +35 (b) −24 (c) +18 (d) −40 (e) −60 (f) +60 (g) −64 (h) +64 **10.** (a) +7 (b) −8 (c) +13 (d) −6 (e) −7 (f) −18 (g) +8 (h) +4

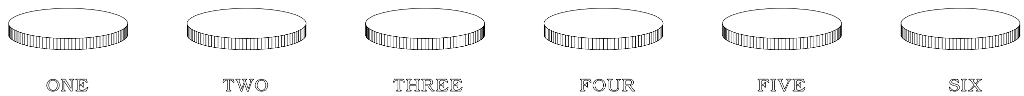
LESSONS

Lesson B2.1 Numbers and Zero

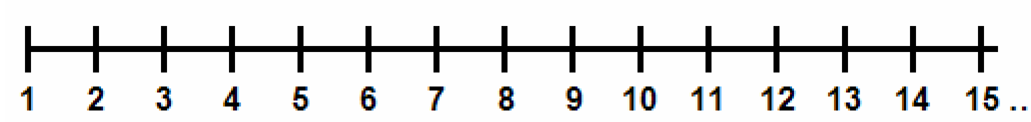
Zero provides a reference point for all numbers.

1. The numbers we use in counting are called NATURAL NUMBERS.

   (a) Numbers 1, 2, 3, etc., have traditionally been used for counting. You count by calling out 1 for the first item, 2 for the second item, 3 for the third item, and so on. These counting numbers are called **natural numbers** because they follow from the natural process of counting.



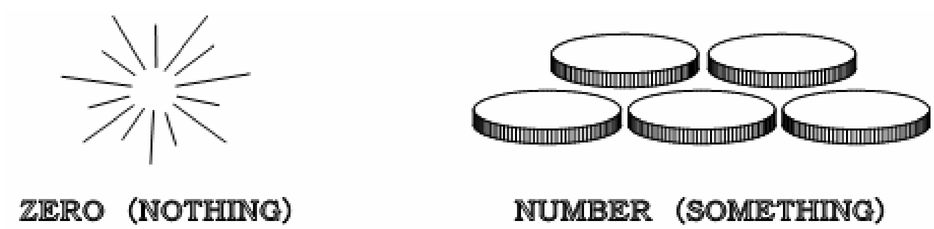
   (b) The natural numbers start from 1. We obtain the next number by increasing a number by 1. Thus, we obtain a sequence of natural numbers that goes on forever. This gives us a Number Line as follows.



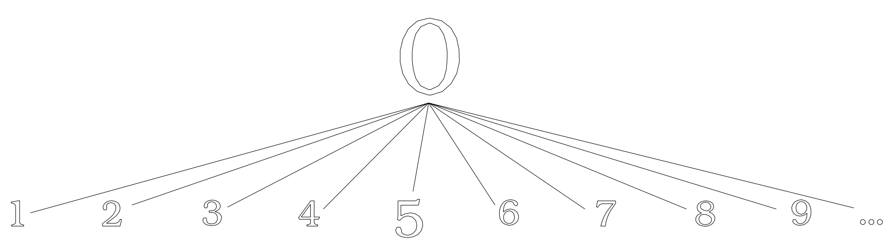
2. Zero is not a natural number because it is not used in counting.

   (a) Reversing the sequence, we decrease a number by 1 to obtain the previous number. When we decrease the smallest natural number 1 by 1 we discover an **"absence of quantity,"** which we call **zero (0)**. For example, when you spend all your money, you have zero money left.

   (b) Zero is not a natural number because, it is not used in counting. The natural numbers, such as, 1, 2, 3, refer to the <u>presence of quantity</u>. Zero refers to an <u>absence of quantity</u>.



   (c) All natural numbers derive their meaning only by comparing them to the background of nothing, or **zero**.



3. On a number scale, we assign a value of **zero** to the point of reference from which other numbers are counted.

(a) People in a room are counted from the reference of an empty room. To the empty room we assign the reference point of "zero people."

(b) Heights of mountains are measured from the reference of sea level. To the sea level we assign the reference point of "zero height."

(c) Years are counted from the reference of the appearance of Christ. To the appearance of Christ we assign the reference point of "year zero."

(d) Temperatures are measured from the reference of freezing water. To freezing water we assign the reference point of "zero temperature."

4. Thus, zero acts as the reference point for wherever natural numbers are used.

   (a) When you are born your age is zero. After 1 year you are a year old. Thus, your age is counted from the time you were born (zero).

   (b) You begin formal education when you are in Kindergarten. Every year you advance by a grade to Grade 1, Grade 2, Grade 3, etc.Thus, the level of your formal education is counted from Kindergarten (zero level).

☺ Exercise B2.1

1. Fill in the blanks
   (a) For the number 55, the next number is __________
   (b) The largest natural number is __________
   (c) For the number 55, the previous number is __________
   (d) The smallest natural number is __________
   (e) For natural number 1, the previous number is __________

2. How is zero uniquely different from all natural numbers? What role does zero play?

3. What reference point would you use as ZERO to get an idea of
   (a) Your height
   (b) The depth of sea
   (c) The distance of a star
   (d) The speed of a moving car

Answer: **1.** 56 **2.** As large as you can think. **3.** 54. **4.** 1 **5.** 0 **6.** Zero is uniquely different because it refers to ***absence*** of quantity, whereas all other numbers refer to ***presence*** of quantity. It acts as the reference point of all other numbers. **7.** (a) Level of your feet (b) The sea level (c) The distance of earth (d) The motion of ground.

Lesson B2.2 Zero and Integers

Numbers may be counted forward or backward from Zero.

1. A physical quantity can be greater or lesser than the point of reference. When a quantity is greater than the reference point we call it **positive**; when it is less we call it **negative**.
   (a) People may be counted not only as present but also as absent from a room.
   (b) Distances exist not only above but also below the sea level.
   (c) Years may be counted not only after but also before the appearance of Christ.
   (d) Temperatures exist not only warmer but also cooler than the freezing water.

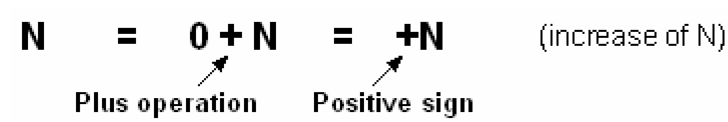
 (01/01/2008)

2. The natural numbers are counted forward from zero. Therefore they are **positive**.

(a) A natural number shows an increase from zero.

0 + 1	=	+1	(increase of 1)
0 + 2	=	+2	(increase of 2)
0 + 3	=	+3	(increase of 3)

(b) A natural number **N** is an increase of N from zero. We do not display the zero but we can leave the **+ sign** in front of the number.



(c) Thus, a number with **+ sign** in front of it, is the same as a number with "no sign."

3. When we count backwards from zero, we get **negative** numbers.

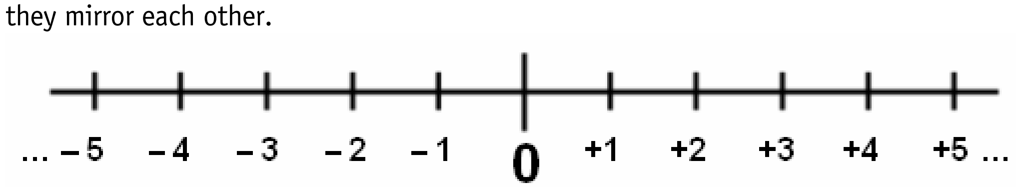
(a) A negative number shows a decrease from zero.

0 − 1	=	−1	(decrease of 1)
0 − 2	=	−2	(decrease of 2)
0 − 3	=	−3	(decrease of 3)

(b) A "shortage" of N is a decrease of N from zero. We do not display the zero but leave the **− sign** in front of the number.

4. Thus, a number with **− sign** in front of it, is opposite to a number with **+ sign**. The positive and negative numbers, together with zero, are referred to as **integers**.

(a) On a number line, the positive and negative numbers appear on opposite sides of zero, and they mirror each other.



(b) The integers have a much larger domain but they preserve the fundamental property of sequence. Adding 1 to an integer gives the next integer.

−5 + 1 = −4; −4 + 1 = −3; −3 + 1 = −2; and so on

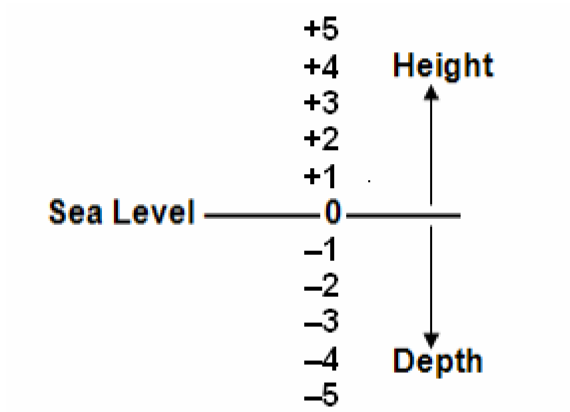
(c) Every integer has exactly one immediate predecessor and exactly one immediate successor, that is, the sequence of integers contains neither a first or a last number. The integers extend indefinitely in either direction from zero.

(d) The integers +2, −3 , +4, −5, +6, −7 shall be arranged on a number line in the following sequence.

−7 −5 −3 +2 +4 +6

5. The point of reference must be the same for quantities that are to be compared to each other. It does not matter what reference is chosen as long as it is fixed for the quantities under comparison.

   (a) The heights of mountains are compared to the depth of seas by using sea level as the common reference point.



☺ Exercise B2.2

1. What is the significance of a number "less than zero"?
2. How would you express
   (a) A shortage of $25 on a scale of money in account.
   (b) A temperature 5 degrees cooler than the freezing water.
   (c) A depth of 50 feet on a scale of heights.
3. What is the purpose of "0" on a number line?
4. Express the following operations with zero as integers.

(a) 0 – 5	(c) 0 + 13	(e) 0 – 37
(b) 0 + 5	(d) 0 – 13	(f) 0 + 37

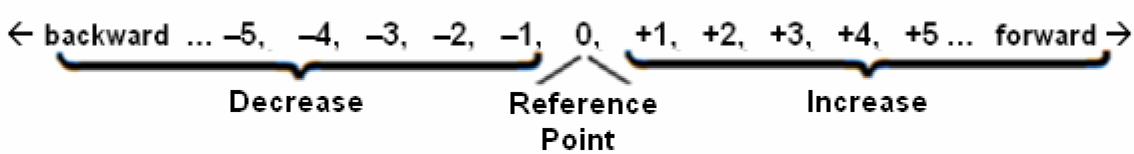
5. Practice ordering of integers by copying and pasting the following link in your browser.

   http://www.bbc.co.uk/skillswise/numbers/wholenumbers/whatarenumbers/negativenumbers/flash1.shtml

Answer: **1.** A number "less than zero" represents a measure from reference point in the opposite direction, such as, a shortage that need to be compensated from elsewhere **2.** (a) – $25 (b) –5 degrees (c) –50 feet **3.** Zero acts as the reference point on a number line. **4.** (a) –5 (b) +5 (c) +13 (d) –13 (e) –37 (f) +3

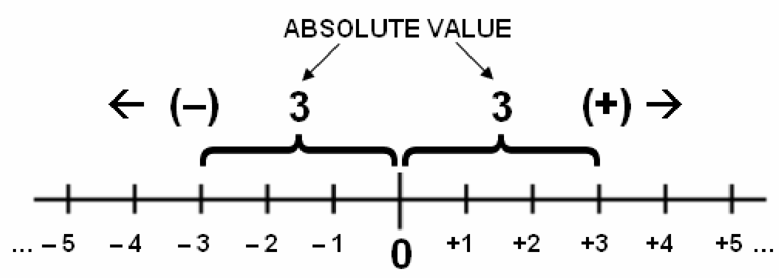
Lesson B2.3 Sign and Absolute value

An integer value is made up of a sign and an absolute value.



1. The sign indicates increase or decrease from the reference point.
   (a) The REFERENCE POINT is represented by “0” (absence of count). It is the starting point of counting. It may be chosen arbitrarily anywhere on a number line.
   (b) Increase is going away from “0” to the right; and it is represented by **+ sign**. Decrease is going away from “0” to the left; and it is represented by **– sign**.
   (c) Quantities having the same sign are said to have **like signs**, and those having different signs, **unlike signs**.

2. Absolute value indicates the amount of incease or decrease.



   (a) On a number line, the absolute value represents the “distance” from “0” no matter what direction it is in.
   (b) The absolute value is represented by two bars on either side of an integer.

$$|+3| = 3$$
$$|-3| = 3$$

   (c) A natural number is written without a sign, and it represents the absolute value. However, it is treated as positive in calculations.
   (d) The absolute value is expressed “without sign” and treated as positive in calculations. The negative of absolute value would be a negative number.

$$|5 - 3| = 2$$
$$|3 - 5| = 2$$
$$-|3 - 5| = -2$$

☺ Exercise B2.3

1. Identify the absolute values below.

   +15, –15, |+15|, 3, –7, |–8|, +9, 12, |–12|, –19,

Answer: **1.** +15, –15, –7, +9 and –19 are integers, All others represent absolute values.

Lesson B2.4 Properties Of Integers

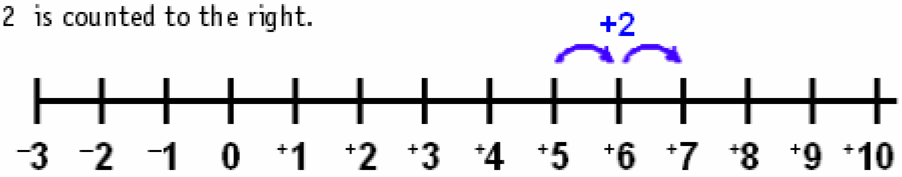
Integers are counted forward or backward depending on their sign.

1. The point of reference, itself is neither negative nor positive. Therefore,

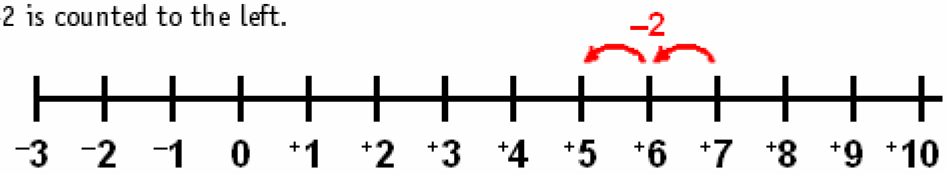
$$+0 = -0 = 0$$

2. From any position on the number line, a positive number is counted to the right, and a negative number is counted to the left.

(a) +2 is counted to the right.



(b) −2 is counted to the left.



(d) +5 counted from −3 will end up at +2.

(e) −8 counted from +4 will end up at −4.

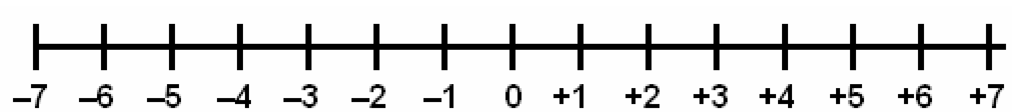
3. A decrease cancels out an equal increase. Therefore, two integers of opposite signs and equal value, when counted together add or subtract nothing. In other words, they nullify each others affect, or cancel each other out.

+2 and −2 = (0 + 2) + (0 − 2) = $0 + \cancel{2} + 0 - \cancel{2}$ = 0

Similarly,

+3 and −3 = 0
+6 and −6 = 0
+9 and −9 = 0

4. Integers may be compared to each other by their location on the number line.



(a) The symbols >, < and = are used to show the relationship between numbers.
">" means, "greater than"
"<" means, "less than"
"=" means, "equal to"

Both **>** and **<** open toward the larger number.

(b) A number is greater than the number to its left.
+2 is to the right of−7 → +2 > −7 (">" means "greater than")
0 is to the right of −5 → 0 > −5
−3 is to the right of −7 → −3 > −7

(c) A number is less than the number to its right.
−3 is to the left of +3 → −3 < +3 ("<" means "less than")
0 is to the left of +5 → 0 < +5
−6 is to the left of −2 → −6 < −2

☺ Exercise B2.4

1. Count as follows from the indicated location. Indicate the location you will reach.

(a) −7 from +5	(d) +8 from +11	(g) −17 from +27	(j) −25 from −22
(b) +5 from −7	(e) −11 from −8	(h) +27 from −27	(k) −22 from +25
(c) −6 from −6	(f) −7 from +7	(i) −13 from −13	(l) +44 from −44

2. Count the following integers one after another from a location on the number line.
   (a) +4 and −4 (b) +15 and −15 (c) +100 and −100 (d) +375 and −375

3. Insert the correct symbol (>, =, or <).

(a) −7 __ +5	(f) +8 __ +11	(k) −17 __ −27	(p) −25 __ −22
(b) +5 __ −7	(g) −11 __ −8	(l) +27 __ +27	(q) −22 __ +25
(c) −6 __ −6	(h) −7 __ +7	(m) −13 __ +13	(r) +44 __ −44
(d) −8 __ +5	(i) −13 __ −13	(n) +13 __ −33	(s) −72 __ −72
(e) −3 __ −8	(j) +1 __ −20	(o) −38 __ +24	(t) +92 __ +93

Answer: **1.** (a) −2 (b) −2 (c) −12 (d) +19 (e) −19 (f) 0 (g) +10 (h) 0 (i) −26 (j) −47 (k) +3 (l) 0 **2.** One returns to the same location as the two equal but opposite integers cancel each other out. **3.** (a) < (b) > (c) = (d) < (e) > (f) < (g) < (h) < (i) = (j) > (k) > (l) = (m) < (n) > (o) < (p) < (q) < (r) > (s) = (t) <

Lesson B2.5 Properties of Signs

POSITIVE affirms the existing characteristic. NEGATIVE indicates the opposite characteristic.

1. Positive of an integer is the same integer.
   Positive of a positive integer is that positive integer: + (+2) → +2
   Positive of a negative integer is that negative integer: + (−2) → −2

2. Negative of an integer is the inverse of that integer.
   Negative of a positive integer is the negative integer: − (+2) → −2
   Negative of a negative integer is the positive integer: − (−2) → +2

3. From the above definition we get the following rule.

   (a) Two consecutive LIKE signs are equivalent to a POSITIVE sign.

   + (+2) → +2 means (+)(+) → (+)

   − (−2) → +2 means (−) (−) → (+)

   (b) Two consecutive UNLIKE signs are equivalent to a NEGATIVE sign.

   + (−2) → −2 means (+)(−) → (−)

   − (+2) → −2 means (−)(+) → (−)

4. In multiplication and division of integers.
   (a) Two LIKE signs are equivalent to a POSITIVE sign
   (b) Two UNLIKE signs are equivalent to a NEGATIVE sign

   ➢ In multiplication

   (−1)(−1) = +1 (+1)(−1) = −1

   (+1)(+1) = +1 (−1)(+1) = −1

- In division

$\frac{(+1)}{(+1)}$	=	+1	$\frac{(+1)}{(-1)}$	=	−1
$\frac{(-1)}{(-1)}$	=	+1	$\frac{(-1)}{(+1)}$	=	−1

☺ Exercise B2.5

1. Simplify.

(a) + (−4)	(d) + (−7)	(g) + (−5)	(j) − (−4)	(m) − (−7)	(p) − (−5)
(b) + (+4)	(e) + (+8)	(h) + (+5)	(k) − (+4)	(n) − (+8)	(q) − (+5)
(c) + (+7)	(f) + (−6)	(i) + (−9)	(l) −(+7)	(o) − (−6)	(r) − (−9)

Answer: **1.** (a) −4 (b) +4 (c) +7 (d) −7 (e) +8 (f) −6 (g) −5 (h) +5 (i) −9 (j) +4 (k) −4 (l) −7 (m) +7
(n) −8 (o) +6 (p) +5 (q) −5 (r) +9

Lesson B2.6 Addition and Subtraction with Integers

We may operate on integers by changing their positive and negative signs to plus and minus operations first and then applying the properties of consecutive signs.

1. Integers represent an increase or decrease from 0.

+2 +5	=	0 + 2 + 5	=	0 + 7	=	+7
−2 +5	=	0 − 2 + 5	=	0 + 3	=	+3
+8 −3	=	0 + 8 − 3	=	0 + 5	=	+5
−8 +3	=	0 − 8 + 3	=	0 − 5	=	−5

2. Integers of **like sign** form a **sum** of the same sign.

+5 +2	=	0 + 5 + 2	=	+7
−5 −2	=	0 − 5 − 2	=	−7

3. Integers of **unlike signs** form a **difference**, with the sign obtained from the larger absolute value.

+3 −8	=	0 + 3 − 8	=	−5
−3 +8	=	0 − 3 + 8	=	+5

4. The COMMUTATIVE PROPERTY applies to the counting together of two integers.

+18 +5	=	+5 +18
+18 −5	=	−5 +18
−18 +5	=	+5 −18
−18 −5	=	−5 −18

5. The ASSOCIATIVE PROPERTY applies to the counting together of three integers. Here, the underlined integers may be counted together first, without changing the order.

+18 <u>+5 +7</u>	=	<u>+18 +5</u> +7
+18 <u>−5 −7</u>	=	<u>+18 −5</u> −7
−18 <u>+5 −7</u>	=	<u>−18 +5</u> −7
−18 <u>−5 +7</u>	=	<u>−18 −5</u> +7

6. When adding or subtracting integers we use the parentheses to isolate the integer value. Then we use the properties of signs to simplify the next consecutive sign. Then we count the resulting integers together.

   (a) The consecutive sign after + remains the same.

(+5) + (+2)	=	+5 +2	=	**+7**
(+5) + (−2)	=	+5 −2	=	**+3**
(−3) + (+8)	=	−3 +8	=	**+5**
(−5) + (−4)	=	−5 −4	=	**−9**

   (b) The consecutive sign after − changes to the opposite.

(+5) − (+2)	=	+5 −2	=	**+3**
(+5) − (−2)	=	+5 +2	=	**+7**
(−3) − (+8)	=	−3 −8	=	**−11**
(−5) − (−4)	=	−5 +4	=	**−1**

7. When integers are added and subtracted, reduce the consecutive signs first. We replace the consecutive signs by a single operation when we remove parentheses.

(+12) − (−7) − (+8) + (−3) + (+2)	**=**	**0 + 12 +7 − 8 − 3 + 2**	**=**	**10**
(−12) − (−7) + (8) − (3) − (−2)	**=**	**0 − 12 + 7 + 8 − 3 + 2**	**=**	**2**

☺ Exercise B2.6

1. Combine the following integers rapidly.

   (a) −7 −5 (b) +5 +7 (c) −2 −5 (d) +6 +8 (e) +9 −9 (f) −9 −6
   (g) −8 −6 (h) +11 +7 (i) −3 −8 (j) +12 +8 (k) −13 −12 (l) +15 −15
   (m) −7 +5 (n) −5 +7 (o) +2 −5 (p) −6 +6 (q) −9 +6 (r) +9 −6
   (s) −8 +6 (t) +7 −7 (u) +3 −8 (v) −12 +8 (w) −13 +13 (x) +15 −10

2. Add the following. Remove parentheses to simplify.

   (a) (−7) + (−5) (b) (+7) + (−5) (c) (+7) + (+5)
   (d) (−7) + (+5) (e) (+9) + (−6) (f) (−9) + (−6)
   (g) (−9) + (+6) (h) (+9) + (+6) (i) (+3) + (−8)
   (j) (+4) + (+9) (k) (−12) + (+7) (i) (−13) + (−8)

3. Subtract the following. Remove parentheses to simplify.

   (a) (−7) − (−5) (b) (+7) − (−5) (c) (+7) − (+5)
   (d) (−7) − (+5) (e) (+9) − (−6) (f) (−9) − (−6)
   (g) (−9) − (+6) (h) (+9) − (+6) (i) (+3) − (−8)
   (j) (+4) − (+9) (k) (−12) − (+7) (i) (−13) − (−8)

4. Count the following integers together.

   (a) +2, −8, +7 (b) +5, −3, −2 (c) +9, −1, −5 (d) +8, −2, −1, −2 (e) +15, −5, −12
   (f) −9, +3, +5, −7 (g) +3, +8, +4, −11 (h) +5, −9, +8, −7 (i) −3, +7, −6, −4, +8 (j) +12, −7, −8, −2, +9
   (k) +6, −5, −7, −12, +3 (l) −5, +3, +9, −8, +2 (m) −16, +8, +7, +5, +5 (n) +14, −6, −8, −7, +5 (o) +7, −6, −5, −11, +4

5. Compute the following.

   (a) (−6) + (+6) + (−7) − (+12) − (−3)
   (b) 16 − (−7) − (+16) + (−3) − 5
   (c) (−9) − 5 − (−9) + (+5) + (−2)
   (d) (+9) − (−12) + (+8) + (−12) − (+7)
   (e) (−3) − (+6) − (−7) − 9 − (−3)
   (f) 6 + (−7) +16 + (−3) − (+5)
   (g) (−4) + 5 − (−9) + (+5) − (−2)

6. Solve the following.
   (a) Find the sum of all integers from - 5 to +5.
   (b) Find the sum of all integers from - 3 to +8
   (c) Find the sum of all integers greater than - 11 and less than - 7.
   (d) If a man earned $342, lost $516 in a bet, earned another $212, and then spent $36 in groceries, how much money does he now have or owe?
   (e) Jeremy bought a record collection for $10, sold it for $15, bought it back for $20, and finally sold it for $25. How much money did Jeremy make or lose?

7. Practice integer operations by copying and pasting these link in your browser.
   http://nlvm.usu.edu/en/nav/frames_asid_161_g_2_t_1.html
   http://www.quia.com/fc/22424.html
   http://www.aplusmath.com/Flashcards/sub-nflash.html

Answer: **1.** (a) –12 (b) +12 (c) –7 (d) +14 (e) 0 (f) –15 (g) –14 (h) +18 (i) –11 (j) +20 (k) –25 (l) 0 (m) –2 (n) +2 (o) –3 (p) 0 (q) –3 (r) +3 (s) –2 (t) 0 (u) –5 (v) –4 (w) 0 (x) +5
2. (a) –12 (b) +2 (c) +12 (d) –2 (e) +3 (f) –15 (g) –3 (h) +15 (i) –5 (j) +13 (k) –5 (l) –21
3. (a) –2 (b) +12 (c) +2 (d) –12 (e) +15 (f) –3 (g) –15 (h) +3 (i) +11 (j) –5 (k) –19 (l) –5
4. (a) +1 (b) 0 (c) +3 (d) +3 (e) –2 (f) –8 (g) +4 (h) –3 (i) +2 (j) +4 (k) –15 (l) +1 (m) +9 (n) –2 (o) –11 **5.** (a) –16 (b) –1 (c) –2 (d) +10 (e) –8 (f) +7 (g) +17 **6.** (a) 0 (b) 30 (c) - 27 (d) +2, or has $2 (f) +10, or made $10

Lesson B2.7 Multiplication and Division with Integers

Multiplication and division of integers may be reduced to a multiplication and division of signs, and a multiplication and division of numbers.

1. Multiplication of integers may be shown by placing integers next to each other in parentheses. No sign is used between the parentheses.

   Multiplication of **–5** by **+2** is written as **(–5)(+2)**
   Multiplication of **+8** by **–3** is written as **(+8)(–3)**
   Multiplication of **–7** by **–4** is written as **(–7)(–4)**

2. The COMMUTATIVE PROPERTY applies in multiplication of two integers.
   (+18)(+5) = (+5)(+18)
   (+18)(–5) = (–5)(+18)
   (–18)(+5) = (+5)(–18)
   (–18)(–5) = (–5)(–18)

8. The ASSOCIATIVE PROPERTY applies in multiplication of three integers. Here, the underlined integers may be multiplied first, without changing the order.

   (+18)<u>(+5)(+7)</u> = <u>(+18)(+5)</u>(+7)
   (+18)<u>(–5)(–7)</u> = <u>(+18)(–5)</u>(–7)
   (–18)<u>(+5)(–7)</u> = <u>(–18)(+5)</u>(–7)
   (–18)<u>(–5)(+7)</u> = <u>(–18)(–5)</u>(+7)

3. An integer is the product of a unit integer (**+1** or **–1**) with its absolute value.
   –5 = (–1)(5)
   +75 = (+1)(75)
   –523 = (–1)(523)

4. We may multiply integers as follows.

   (a) Separately multiply the unit integers and absolute values.

   (−5)(+2) = (−1)(+1) (5)(2) = (−1)(10) = −10
   (−7)(−3) = (−1)(−1) (7)(3) = (+1)(21) = +21

   (b) Use the unit integers to actually determine the sign of the product of the absolute values.

   (+2)(+5)(−3) = (+)(+)(−) (2)(5)(3) = −30
   (+3)(−4)(−6) = (+)(−)(−) (3)(4)(6) = +72

5. We may divide integers as follows.

   (a) Separately divide the unit integers and absolute values.

$$\frac{(-12)}{(+3)} = \frac{(-1)(12)}{(+1)\ (3)} = (-1)(4) = -4$$

$$\frac{(-27)}{(-9)} = \frac{(-1)(27)}{(-1)\ (9)} = (+1)(3) = +3$$

   (b) Use division as "multiplication by the inverse," and use the unit integers to actually determine the sign of the product as in multiplication above. *Note that slash (/) is used to show the inverse being a symbol for division.*

   (+12)(+1/2)(−1/3) = (+)(+)(−) (12)(1/2)(1/3) = −2
   (+72)(−1/6)(−1/4) = (+)(−)(−) (72)(1/6)(1/4) = +3

☺ Exercise B2.7

1. What does the following notation mean?

   (a) (+32) (−21) (b) (−17)(− 42) (c) (21)(38)

2. Write the following integers as multiples of unit integers.

(a)	−4	(d)	−19	(g)	−272
(b)	+4	(e)	+48	(h)	+153
(c)	+6	(f)	−71	(i)	−345

3. Multiply the following integers.

(a)	(−7)(−5)	(d)	(−7)(+5)	(g)	(−9)(+6)	(j)	(+4)(+5)(−2)
(b)	(+7)(−5)	(e)	(+9)(−6)	(h)	(+9)(+6)	(k)	(−2)(−7)(+5)
(c)	(+7)(+5)	(f)	(−9)(−6)	(i)	(+3)(−8)	(l)	(−3)(−4)(−6)

4. Divide the following integers. *Note that slash (/) is used as a symbol for division.*

(a)	(−10)/(−5)	(d)	(−10)/(+5)	(g)	(−9)/(+3)	(j)	(+45)/(−9)
(b)	(+10)/(−5)	(e)	(+9)/(−3)	(h)	(+9)/(+3)	(k)	(−91)/(+13)
(c)	(+10)/(+5)	(f)	(−9)/(−3)	(i)	(+48)/(−8)	(l)	(−39)/(−3)

Answer: **1.** (a) Multiply +32 by −21 (b) Multiply −17 by −42 (c) Multiply 21 by 38 **2.** (a) (−1)(4) (b) (+1)(4) (c) (+1)(6) (d) (−1)(19) (e) (+1)(48) (f) (−1)(71) (g) (−1)(272) (h) (+1)(153) (i) (−1)(345) **3.** (a) +35 (b) −35 (c) +35 (d) −35 (e) −54 (f) +54 (g) −54 (h) +54 (i) −24 (j) −40 (k) +70 (l) −72 **4.** (a) +2 (b) −2 (c) +2 (d) −2 (e) −3 (f) +3 (g) −3 (h) +3 (i) −6 (j) −5 (k) −7 (l) +13

SUMMARY

Mathematics of integers appears to be quite troublesome to most students. But when we look at it as arithmetic with increase and decrease (from zero), it becomes easy to grasp.

Confusion takes place because “plus” and “minus,” which represent operations between two numbers, are also used to show a number as “positive” or “’negative.” This confusion is clarified when integers are defined as referenced from zero. This allows us to convert integers into operations with natural numbers, and vice versa.

Further confusion arises when “plus” and “minus” operate on “positive” and “negative” numbers, giving consecutive signs. However, once we understand that LIKE consecutive signs produce a positive number, and UNLIKE consecutive signs produce a negative number, the operations are greatly simplified.

DIAGNOSTIC TEST

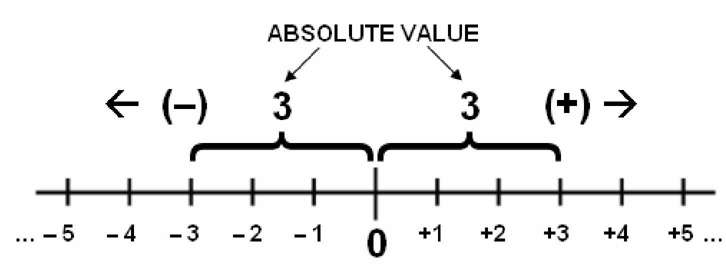
1. Convert the following operations with natural numbers to integers.
   (a) 0 – 7 → ___ (b) 0 + 5 → ___ (c) 0 – 35 → ___ (d) 0 + 56 → ___
2. Convert the following integers to operations with natural numbers.
   (a) –9 → ___ (b) +8 → ___ (c) –23 → ___ (d) +47 → ___
3. Draw a number line and show where the following numbers would appear:
   +15, 0, –15, –11, +3, –7, 8, +9, 20,
4. Insert the correct symbol (>, =, or <)
   (a) –7 __ +5 (b) +2 __ –5 (c) –3 __ –8 (d) +8 __ +11 (e) –9 __ +9
5. What number will you reach by counting on a number line
   (a) +7 from +5 (b) +8 from –3 (c) –7 from +5 (d) –8 from –3
6. Count the following integers together.
   (a) +2, –8 and +7 (b) –9, +3, +5 and –7 (c) +6, –5, –7, –12 and +3
7. Add the following
   (a) (–7) + (–5) (b) (–7) + (+5) (c) (–9) + (+6) (d) (+4) + (+9)
8. Subtract the following
   (a) (–7) – (–5) (b) (–7) – (+5) (c) (+9) – (–6) (d) (+4) – (+9)
9. Multiply the following.
   (c) (–7)(–5) (c) (+6)(+3) (e) (+3)(–4)(+5) (g) (–8)(–2)(–4)
   (d) (–4)(+6) (d) (+8)(–5) (f) (–3)(+4)(–5) (h) (+8)(+2)(+4)
10. Divide the following.
   (c) (–35) / (–5) (c) (+39) / (+3) (e) (+63) / (–9) (g) (–56) / (–7)
   (d) (–48) / (+6) (d) (+42) / (–7) (f) (–72) / (+4) (h) (+36) / (+9)

Answer: **1.** (a) –7 (b) +5 (c) –35 (d) +56 **2.** (a) 0 – 9 (b) 0 + 8 (c) 0 – 23 (d) 0 + 47 **4.** (a) < (b) > (c) > (d) < (e) < **5.** (a) +12 (b) +5 (c) –2 (d) –11 **6.** (a) +1 (b) –8 (c) –15 **7.** (a) –12 (b) –2 (c) –3 (d) +13 **8.** (a) –2 (b) –12 (c) +15 (d) –5 **9.** (a) +35 (b) –24 (c) +18 (d) –40 (e) –60 (f) +60 (g) –64 (h) +64 **10.** (a) +7 (b) –8 (c) +13 (d) –6 (e) –7 (f) –18 (g) +8 (h) +4

GLOSSARY

[For additional words refer to the glossaries at the end of earlier Milestones]

Absolute Value — The **Absolute value** is the "size" of a value irrespective of the "direction" in which it is being counted. On a number line, the absolute value represents the distance from 0.



The absolute value of +3: $|+3| = 3$
The absolute value of –3: $|-3| = 3$

Additive inverse — An **Additive inverse** of an integer is the integer of opposite sign.

Greater than — The **"greater than" sign (>)** expresses, "The first number or expression is greater than the second number or expression."

Integers — When we consider both positive and negative numbers together with zero as their reference point we have a set of INTEGERS.

Less than — The **"Less than" sign (>)** expresses, "The first number or expression is less than the second number or expression."

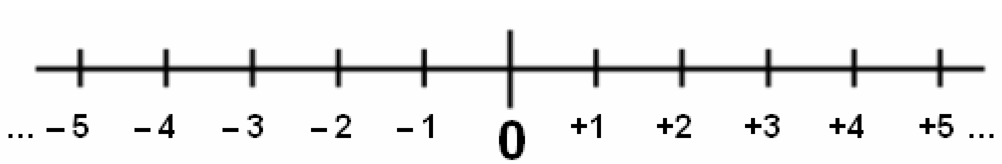
Like Signs — Numbers having the same sign are said to have *like* signs.

+A and +B have like signs
–A and –B have like signs

Natural Numbers — The numbers we use in counting are called **natural numbers.** The word NATURAL comes from a Latin word, *nasci*, which means, *"be born."* The smallest natural number is **1.** The largest natural number is as large as you can think of.

Negative Number — A **negative number** is a number with a **'–'** sign in front of it as in **–8.** The **'–'** sign in front of a number shows that the number is that much less than 0. Also see **Number Line.**

Number Line — The **number line** shows numbers by their "distance" from 0 as follows.



The numbers counted to the right from 0 are positive numbers

+N = 0 + N

The numbers counted to the left from 0 are negative numbers

$$-N = 0 - N$$

Positive Number — A **positive number** is a number with a plus **'+'** sign in front of it as in **+8.** The **'+'** sign in front of a number shows that the number is that much greater than 0. Also see **Number Line.**

Unlike Signs — Number Quantities having different signs are said to have *unlike* signs.
–A and +B have unlike signs
+A and –B have unlike signs

Whole Numbers — When we consider natural numbers together with zero as their reference point we have whole numbers. The word WHOLE came from a prehistoric word meaning *'undamaged.'*