

MATH MILESTONES # A4

MULTIPLICATION

The word, **milestone**, means “a point at which a significant change occurs.” A Math Milestone refers to a significant point in the understanding of mathematics.

To reach this milestone one should be able to multiply single and double-digit numbers mentally without effort.

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A calculator shall be required to check the answers arrived at through mental math.

Please consult the [Glossary](#) supplied with this Milestone for mathematical terms. Consult a regular dictionary at www.dictionary.com for general English words that one does not understand fully.

You may start with the Diagnostic Test on the next page to assess your proficiency on this milestone. Then continue with the lessons with special attention to those, which address the weak areas.

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DIAGNOSTIC TEST

1. Tell without adding which of the following is equal and why?

- (a) $2 + 2 + 2 = 3 + 3$
- (b) $5 + 5 + 5 = 3 + 3 + 3 + 3 + 3$
- (c) $7 + 7 + 7 = 3 + 3 + 3 + 3 + 3 + 3$
- (d) $9 + 9 + 9 = 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3$

2. Multiply the following mentally.

- | | | |
|------------------|------------------|------------------|
| (a) 2×9 | (d) 8×3 | (g) 9×5 |
| (b) 5×8 | (e) 4×7 | (h) 8×6 |
| (c) 9×6 | (f) 7×9 | (i) 7×6 |

3. Compute the following mentally.

- | | | |
|----------------------|-------------------|--------------------|
| (a) 375×0 | (g) 32×5 | (m) 70×40 |
| (b) 375×1 | (h) 17×5 | (n) 7×13 |
| (c) 125×10 | (i) 18×2 | (o) 8×37 |
| (d) 16×100 | (j) 14×4 | (p) 6×35 |
| (e) 24×1000 | (k) 13×8 | (q) 30×7 |
| (f) 6×80 | (l) 45×9 | (r) 45×99 |

4. Multiply the following on paper by columns.

- | | | |
|-------------------|---------------------|------------------------|
| (a) 43×2 | (c) 123×13 | (e) 507×203 |
| (b) 52×3 | (d) 175×63 | (f) 3005×3005 |

5. Solve the following word problems.

- (a) What is the cost of 65 printers, if each printer costs \$75?
- (b) If 4 men can paint a house in 3 days, how many men will it take to paint the house in 1 day?
- (c) Jan needs drapes that are 3 yards high and 5 yards long to cover a picture window in her home. If she pays \$7 a square yard for the material she likes, what will the drapes cost?
- (d) A farmer bought 24 sheep, at \$5 a head; 36 hogs at \$14 a head; and 9 cows at \$45 a head. When he sold them all he lost \$276. For how much did he sell them?

Answer: 1. (a) 2×3 and 3×2 are equal (b) 5×3 and 3×5 are equal (c) 7×3 and 3×7 are not equal (d) 9×3 and 3×9 are equal 2. (a) 18 (b) 40 (c) 54 (d) 24 (e) 28 (f) 63 (g) 45 (h) 48 (i) 42 (j) 56 (k) 104 (l) 104 (m) 2800 (n) 91 (o) 296 (p) 210 (q) 210 (r) 4455 4. (a) 86 (b) 156 (c) 1599 (d) 11025 (e) 102921 (f) 9030025 5. (a) \$4875 (b) 12 men (c) \$105 (d) \$753

LESSONS

Lesson A4.1 Repeated Addition

MULTIPLICATION is adding a quantity repeatedly to itself. The outcome is called the **PRODUCT**.

1. There are 10 baskets, each containing 6 apples. To find out how many apples there are, you count 6 apples ten times for the 10 baskets. You get a total of 60 apples. We say that the product of 6 counted 10 times is 60.

$$6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 = 60 \text{ apples}$$

2. There are 7 families in a village, each having 3 pets. To find out how many pets there are, you count 3 pets seven times for the 7 families. You get a total of 21 pets. We say that the product of 3 counted 7 times is 21.

$$3 + 3 + 3 + 3 + 3 + 3 + 3 = 21 \text{ pets}$$

3. When 5 books are sold at \$6 each, the money received is. We say that the product of 6 counted 5 times is 30.

$$\begin{array}{c} \$6 + \$6 + \$6 + \$6 + \$6 = \$30 \\ \leftarrow \text{count 5 times} \rightarrow \end{array}$$

☺ Exercise A4.1

1. Answer the following.
 - (a) You collect \$5 from each of your 4 friends for a pizza party. How much money did you collect?
 - (b) There are 12 inches in a foot. How many inches are there in 5 feet?
 - (c) Each mango costs \$3. You bought 8 mangoes. How much did it cost you?
2. Find the product when
 - (a) 3 is added repeatedly 6 times
 - (b) 6 is added repeatedly 3 times
 - (c) 5 is added repeatedly 8 times
 - (d) 7 is added repeatedly 4 times
 - (e) 9 is added repeatedly 5 times
 - (f) 8 is added repeatedly 6 times

Answer: 1. (a) \$20 (b) 60 inches (c) \$24 2. (a) 18 (b) 18 (c) 40 (d) 28 (e) 45 (f) 48

Lesson A4.2 Terms and Definitions

Multiplication is "repeated addition" a number of times.

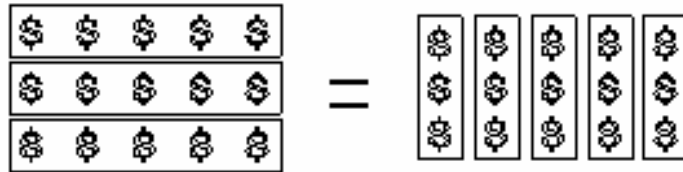
1. We write multiplication with a multiplication sign (x) between the numbers to be multiplied. For example, if we are multiplying 6 five times, we may express it as follows.

$$6 \times 5 = 30$$

- (a) "x" is the **sign** for multiplication; "=" is the **equals sign**.
- (b) We read the above expression as, "**6 times 5 equals 30.**"
- (c) The numbers to be multiplied, 6 and 5, are called **factors** (part of multiplication).

(d) The outcome of multiplication, 30, is called the **product**.

2. We may multiply two numbers in any order. We may have 3 rows of 5 dollars each, or have 5 columns of 3 dollars each. The product is the same in either case.



$$\begin{array}{rcl} & 5 + 5 + 5 & = 15 \\ & 3 + 3 + 3 + 3 + 3 & = 15 \\ \text{Therefore,} & 5 + 5 + 5 & = 3 + 3 + 3 + 3 + 3 \end{array}$$

Or,
$$5 \times 3 = 3 \times 5$$

This is called the **Commutative Property Of Multiplication**. The word COMMUTE means “to change the order or arrangement of.”

3. We may say that, 6 counted 5 times is the same as 5 counted 6 times.

$$\begin{array}{rcl} & 6 + 6 + 6 + 6 + 6 & = 5 + 5 + 5 + 5 + 5 + 5 \\ \text{Because,} & 6 \times 5 & = 5 \times 6 \end{array}$$

Therefore, to multiply, we may start with the larger number and count as many more as the smaller number.

$$3 \times 15 = 15 \times 3 = 15 + 15 + 15 = 45$$

4. Some simple word problems require multiplication only:

(a) *At \$7 a basket, what will 4 baskets of apples cost?*

This is clearly a multiplication problem. We multiply 7×4 , getting a product of 28.

Therefore, total cost would be \$28.

(b) *On a chess-board are 8 rows of squares, and 8 squares in each row: how many squares on the board?*

This is also a multiplication problem. We multiply 8×8 . The product is 64. Therefore, there are 64 squares on the board.

😊 Exercise A4.2

- Write down “4 times 3 equals 12” using mathematical symbols.
- Identify the factors and product in the following.
 - $9 \times 3 = 27$
 - $8 \times 7 = 56$
 - $5 \times 23 = 115$
- Can you tell without adding if the following are equal? Correct any errors.
 - $2 + 2 + 2 = 3 + 3$
 - $5 + 5 + 5 = 3 + 3 + 3 + 3 + 3$
 - $7 + 7 + 7 = 3 + 3 + 3 + 3 + 3 + 3$
 - $4 + 4 + 4 = 3 + 3 + 3 + 3$
 - $9 + 9 + 9 = 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3$

4. Multiply by repeated addition.
- (a) 8×3 (d) 2×35 (g) 21×5
 (b) 4×7 (e) 3×25 (h) 16×6
 (c) 9×5 (f) 4×16 (i) 23×3
5. Solve the following using multiplication.
- (a) Abacus has 10 wires. Each wire has 10 beads. How many beads are there in all?
 (b) John plants 4 trees every month. How many trees did he plant in a year?
 (c) Elli works 8 hours every day. She goes to work 5 days a week. How many hours does she work in a week?

Answer: 1. $4 \times 3 = 12$. Factors are on the left of the "=" sign, and product is on the right.
 2. Start with the larger number. (a) 2×3 and 3×2 , equal (b) 5×3 and 3×5 , equal (c) 7×3 and 3×7 , not equal (d) 4×3 and 3×4 , equal (e) 9×3 and 3×9 , not equal (f) 5 . (g) 24 (h) 28 (i) 45 (j) 70 (k) 75 (l) 64 (m) 105 (n) 96 (o) 69 (p) 100 beads (q) 48 trees (r) 40 hours

☺ **Practice #1**

1. Count on your ten fingers "by 1," "by 2," "by 3," etc.
- (a) Count on your ten fingers "by 1" as, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. Write down these counts in a column as in the table below.
- (b) Count on your ten fingers "by 2" (repeatedly adding 2) as 2, 4, 6, 8, and so on. Write down these counts in a column next to the first one as shown below.
- (c) Similarly, count on your ten fingers, respectively, "by 3," "by 4," "by 5," etc. up to "by 12." using **repeated addition**. Write down the counts in columns, and then check them against the table below. *NOTE: You may go up to "by 20" for further practice of repeated addition.*

Repeated Addition																			
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200

2. Note that this table of "repeated addition" is also a **multiplication table**. A product, such as, 7×6 is equal to "the number in 7th column and 6th row" = 42.
3. Practice creating this table until you can do so quickly.

Lesson A4.3 Mental Multiplication

Patterns in multiplication lead to simple shortcuts.

1. When you multiply a number by 0, the product is 0.

(a) 0 multiplied any number of times will be 0, because adding zero repeatedly adds nothing.

$$0 \times 6 = 0 + 0 + 0 + 0 + 0 + 0 = 0$$

(b) To multiply a number by 0 means that the number is not counted at all.

$$6 \times 0 = (\text{6 is not counted at all}) = 0$$

(c) Therefore, for any number N,

$$\mathbf{0 \times N = N \times 0 = 0}$$

2. When you multiply a number by 1, the product is the number itself.

(a) To multiply 1 by a number is to count 1 as many times as the number.

$$1 \times 6 = 1 + 1 + 1 + 1 + 1 + 1 = 6$$

(b) A number multiplied 1 time means that the number is counted once only.

$$6 \times 1 = (\text{6 is counted once}) = 6$$

(c) Therefore, for any number N,

$$\mathbf{1 \times N = N \times 1 = N}$$

3. When you multiply a number by 10, the product is the number followed by a 0.

(a) To multiply by 10 is to move each digit to the next higher place value. This leaves a 0 in ONES place.

$$25 \times 10 = 250$$

(b) Therefore, for any number N,

$$\mathbf{N \times 10 = N'0'}$$

When you multiply a number by 100, the product is the number followed by two 0's.

$$13 \times 100 = 1300$$

When you multiply a number by 1000, the product is the number followed by three 0's.

$$75 \times 1000 = 75000 \quad (\text{75 thousands})$$

4. To multiply a number by 2, double the number.

(a) Doubling the number is adding the number to itself.

$$7 \times 2 = 7 + 7$$

(b) Therefore, for any number N,

$$\mathbf{N \times 2 = N + N}$$

To multiply a number by 4, double the number and then double it again.

$$7 \rightarrow 14 \rightarrow 28$$

To multiply a number by 8, double the number three times in succession.

$$8 \rightarrow 16 \rightarrow 32 \rightarrow 64$$

5. When you multiply a number by 5, the product is half of when you multiply by 10.

(a) When a number is EVEN, take half the number and place '0' after it.

$$6 \times 5 \rightarrow \text{Take half of } 6 \text{ (3) and place '0' } \rightarrow 30$$

$$14 \times 5 \rightarrow \text{Take half of } 14 \text{ (7) and place '0' } \rightarrow 70$$

(b) When a number is ODD, take the HALF of the previous number and place '5' after it.

$$9 \times 5 \rightarrow \text{Take half of } 9-1 \text{ (4) and place '5' } \rightarrow 45$$

$$17 \times 5 \rightarrow \text{Take half of } 17-1 \text{ (8) and place '5' } \rightarrow 85$$

(c) Therefore, for any number N,

When N is even, $N \times 5 = \text{half of } N \text{ with '0'}$

When N is odd, $N \times 5 = \text{half of } N-1 \text{ with '5'}$

6. To multiply a number by 9, is to add the number to itself one time less than 10 times.

(a) This is same as multiplying by 10 and then subtracting the number.

$$7 \times 9 = (7 \times 10) - 7 = 63$$

$$15 \times 9 = (15 \times 10) - 15 = 135$$

(b) Therefore, for any number N,

$$N \times 9 = (N \times 10) - N$$

☺ Exercise A4.3

1. Multiply the following using the formulas above.

(a) 3×10

(d) 8×1000

(g) 9×100

(j) $16,437 \times 0$

(b) 14×1

(e) 28×100

(h) 23×10

(k) 231×1000

(c) 125×0

(f) 327×10

(i) 385×1

(l) $32,862 \times 1$

Have another person call out numbers randomly. Multiply that number by 0, 1, 10, 100 and 1000 respectively, until you feel confident. *If possible, do this exercise on a turn about basis.*

2. Multiply the following using the formulas above.

(a) 11×2

(d) 8×4

(g) 7×8

(j) 16×8

(b) 14×2

(e) 12×4

(h) 15×8

(k) 18×4

(c) 18×2

(f) 17×4

(i) 13×8

(l) 19×2

Have another person call out numbers up to 20 randomly. Multiply that number by 2, 4 and 8 respectively, until you feel confident. *If possible, do this exercise on a turn about basis.*

3. Multiply the following using the formulas above.

(a) 6×5

(d) 9×5

(g) 48×5

(b) 12×5

(e) 15×5

(h) 53×5

(c) 28×5

(f) 25×5

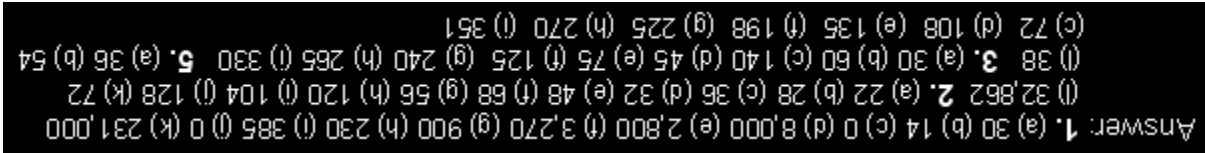
(i) 66×5

Have another person call out double-digit numbers randomly. Multiply that number by 5 and check your answer on a calculator. Do this until you can give three correct answers in a row. *If possible, do this exercise on a turn about basis.*

4. Practice multiplying single-digit numbers by 9 using the formula above. Check your answer on a calculator.

5. Multiply the following using the formula above.
- | | | |
|------------------|-------------------|-------------------|
| (a) 4×9 | (d) 12×9 | (g) 25×9 |
| (b) 6×9 | (e) 15×9 | (h) 30×9 |
| (c) 8×9 | (f) 22×9 | (i) 39×9 |

Have another person call out numbers up to 20 randomly. Multiply that number by 9 and check your answer on a calculator. Do this until you can give three correct answers in a row. *If possible, do this exercise on a turn about basis.*



Lesson A4.4 Properties of Multiplication

We may distribute multiplication by place values.

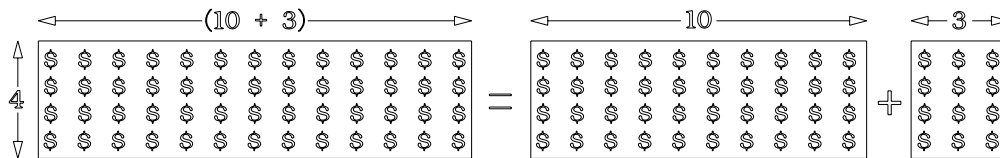
1. To multiply by a number greater than 10, one may distribute the multiplication between TENS and ONES.

(a) We may distribute multiplication (repeated addition) over TENS and ONES as follows.

$$\begin{aligned}
 4 \times 13 &= 4 + 4 + 4 + \dots \text{ (13 times)} \\
 &= 4 + 4 + 4 + \dots \text{ (10 times)} + 4 + 4 + 4 \text{ (3 times)} \\
 &= (4 \times 10) + (4 \times 3)
 \end{aligned}$$

We may write this as follows.

$$4 \times 13 = 4 \times (10 + 3) = (4 \times 10) + (4 \times 3) = 40 + 12 = 52$$



This is called the **Distributive Property of Multiplication**. This is the most used property in mathematics.

(b) Therefore, we have

$$\begin{aligned}
 7 \times 13 &= 7 \times (10 + 3) = 7 \times 10 + 7 \times 3 = 70 + 21 = 91 \\
 5 \times 25 &= 5 \times (20 + 5) = 5 \times 20 + 5 \times 5 = 100 + 25 = 125 \\
 8 \times 37 &= 8 \times (30 + 7) = 8 \times 30 + 8 \times 7 = 240 + 56 = 296
 \end{aligned}$$

(c) For any three number **A**, **B**, and **C**,

$$\mathbf{A \times (B + C) = (A \times B) + (A \times C)}$$

2. When multiplying multiples of TEN, we may detach and attach trailing 0's.

(a) We may multiply 7 by 30 as follows.

$$7 \times 30 = 7 \times (3 \times 10) = (7 \times 3) \times 10 = 21 \times 10 = 210$$

(b) We may multiply 40 by 60 as follows.

$$40 \times 60 = (4 \times 10) \times (6 \times 10) = (4 \times 6) \times (10 \times 10) = 24 \times 100 = 2400$$

(c) Therefore, we may detach the trailing zero in initial multiplication, and then attach it later to the product.

$$\begin{array}{l} 3 \times 90 \rightarrow 3 \times 9 \text{ (detach 0)} \rightarrow 27 \text{ (attach 0)} \rightarrow 270 \\ 60 \times 80 \rightarrow 6 \times 8 \text{ (detach 00)} \rightarrow 48 \text{ (attach 00)} \rightarrow 4800 \end{array}$$

3. From the above, we can see that we may group three numbers in two different ways, without changing their sequence. The outcome is the same.

$$7 \times 30 = 7 \times (3 \times 10) = (7 \times 3) \times 10 = 210$$

This is called the **Associative Property of Multiplication**. The meaning of the word ASSOCIATE is, "to group together."

$$A \times (B \times C) = (A \times B) \times C$$

☺ Exercise A4.4

1. Multiply the following using the **Distributive Property**.

- | | | |
|-------------------|-------------------|-------------------|
| (a) 3×14 | (g) 5×39 | (m) 3×77 |
| (b) 7×23 | (h) 8×47 | (n) 8×53 |
| (c) 6×17 | (i) 7×58 | (o) 4×93 |
| (d) 4×35 | (j) 9×23 | (p) 2×67 |
| (e) 2×83 | (k) 6×68 | (q) 6×34 |
| (f) 3×38 | (l) 2×59 | (r) 5×88 |

Call out a single-digit numbers, and a double-digit number up to 16, for the student to multiply mentally. Check the answer on a calculator, or from the table below. *Do this exercise on a turn about basis, if possible.*

2. Multiply the following using the **Associative Property**.

- | | | |
|--------------------|--------------------|------------------------|
| (a) 6×30 | (d) 17×30 | (g) 40×40 |
| (b) 30×50 | (e) 90×20 | (h) 500×20 |
| (c) 20×80 | (f) 3×330 | (i) 3000×4000 |

Have another person call out multiples of TEN to multiply. Repeat this until you can multiply confidently. Check your answer on a calculator. *Do this exercise on a turn about basis, if possible.*

3. Write the multiplication table for the numbers 1 to 20.

Answer: 1. (a) 42 (b) 161 (c) 102 (d) 140 (e) 166 (f) 114 (g) 195 (h) 376 (i) 406 (j) 207 (k) 408 (l) 118 (m) 231 (n) 424 (o) 372 (p) 134 (q) 204 (r) 440 2. (a) 180 (b) 1500 (c) 1600 (d) 510 (e) 1800 (f) 990 (g) 1600 (h) 10,000 (i) 12,000,000

Lesson A4.5 Multiplying By Columns

We may multiply numbers by arranging their digits in columns by their place values.

1. This is just another way of multiplying using the **Distribution Property**.

Multiply 23 x 2 using columns.

The mental steps are:

- (a) For 1's: 2 times 3 equal 6 ones.
 (b) For 10's: 2 times 2 equal 4 tens.
 (c) The product is 46.

$$\begin{aligned} 23 \times 2 &= (20 + 3) \times 2 \\ &= (20 \times 2) + (3 \times 2) \\ &= 40 + 6 \\ &= 46 \end{aligned}$$

	tens	ones	
	2	3	factor
x	2	2	factor
	4	6	product

2. One multiplies by columns from right to left. When the product in a column is more than ten then one carries the "tens" over to the column on the left.

Multiply 23 by 4

- (a) For 1's: 4 times 3 equals 12; put down 2 ones; carry over 1 ten;
 (b) For 10's: 4 times 2 equal 8, plus 1 carried over, equals 9; put down 9 tens.
 (c) The product is 92.

	1	carry over
	2	3
x	4	
	9	2

Multiply 69 by 7

- (a) For 1's: 7 times 9 equal 63; put down 3; carry over 6 tens;
 (b) For 10's: 7 times 6 equal 42, plus 6 carried over, equal 48 tens; put down 8; carry over 4 hundreds;
 (c) For 100's: No digit in hundred's column; carried is 4 hundreds; put down 4.
 (d) The product is 483.

	4	6
	6	9
x	7	
	4	8
	4	8

3. To multiply by double-digit numbers, obtain partial products for TENS and ONES. The product is the sum of the partial products.

Multiply 123 by 13

- (a) First multiply 123 by 3 ones, and write the partial product 369.
 (b) Next multiply 123 by 1 ten, and write the partial product 1230.
 (c) Add the two partial products together to get the final product 1599.

	1	2	3		
	x	1	3		
	3	6	9	(123 x 3 ones)	
	1	2	3	0	(123 x 1 ten)
	1	5	9	9	

It is customary to omit the final zero of the second line. We write the partial product from right to left, starting from the same column as the digit in the second factor (multiplier).

- (a) Multiply 123 by 3 ones. The digit '9' of the partial product 369 starts from the column of 3 (in 13).
 (b) Multiply 123 by 1 ten. The digit '3' of the partial product 123 starts from the column of 1 (in 13).

	1	2	3	
	x	1	3	
	3	6	9	(123 x 3 ones)
	1	2	3	(123 x 1 ten)
	1	5	9	9

Multiply 376 by 24

Note that there may be two sets of carry-overs, one for each digit of the second factor (24), as shown here.

Do this multiplication on paper and then check out the product **9,024** on a calculator.

$$\begin{array}{r}
 \begin{array}{r}
 1 \quad 1 \quad \text{carry-overs for 2} \\
 1 \quad 3 \quad 2 \quad \text{carry-overs for 4} \\
 3 \quad 7 \quad 6 \\
 \times 2 \quad 4 \\
 \hline
 1 \quad 5 \quad 0 \quad 4 \quad (376 \times 4 \text{ ones}) \\
 7 \quad 5 \quad 2 \quad \quad (376 \times 2 \text{ tens}) \\
 \hline
 9 \quad 0 \quad 2 \quad 4
 \end{array}
 \end{array}$$

Multiply 427 by 35.

In most cases carry-overs can be kept in mind when multiplying, and need not be written.

Do this multiplication on paper and then check out the product **14,945** on a calculator.

$$\begin{array}{r}
 \begin{array}{r}
 \quad \quad 4 \quad 2 \quad 7 \\
 \quad \quad \times \quad 3 \quad 5 \\
 \hline
 \quad \quad 2 \quad 1 \quad 3 \quad 5 \\
 1 \quad 2 \quad 8 \quad 1 \\
 \hline
 1 \quad 4 \quad 9 \quad 4 \quad 5
 \end{array}
 \end{array}$$

☺ **Exercise A4.5**

1. Multiply the following numbers by columns.

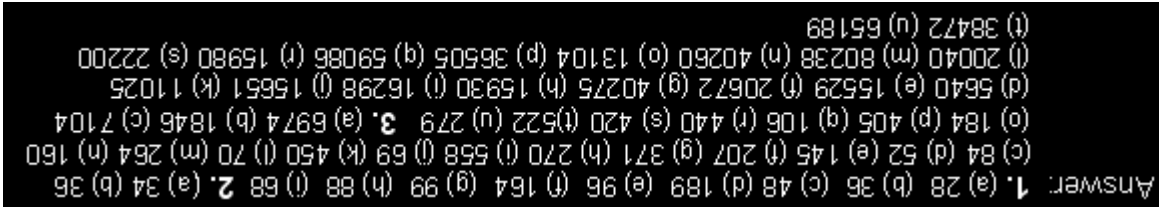
- | | | |
|------------|------------|------------|
| (a) 14 x 2 | (d) 21 x 9 | (g) 33 x 3 |
| (b) 12 x 3 | (e) 32 x 3 | (h) 44 x 2 |
| (c) 12 x 4 | (f) 41 x 4 | (i) 34 x 2 |

2. Multiply the following numbers by columns.

- | | | |
|--------------------|--------------------|--------------------|
| (a) 17 x 2 = _____ | (h) 54 x 5 = _____ | (o) 46 x 4 = _____ |
| (b) 12 x 3 = _____ | (i) 62 x 9 = _____ | (p) 45 x 9 = _____ |
| (c) 12 x 7 = _____ | (j) 23 x 3 = _____ | (q) 53 x 2 = _____ |
| (d) 13 x 4 = _____ | (k) 75 x 6 = _____ | (r) 88 x 5 = _____ |
| (e) 29 x 5 = _____ | (l) 35 x 2 = _____ | (s) 70 x 6 = _____ |
| (f) 23 x 9 = _____ | (m) 33 x 8 = _____ | (t) 87 x 6 = _____ |
| (g) 53 x 7 = _____ | (n) 32 x 5 = _____ | (u) 93 x 3 = _____ |

3. Multiply the following by columns.

- | | | |
|----------------------|----------------------|----------------------|
| (a) 317 x 22 = _____ | (h) 354 x 45 = _____ | (o) 546 x 24 = _____ |
| (b) 142 x 13 = _____ | (i) 562 x 29 = _____ | (p) 745 x 49 = _____ |
| (c) 222 x 32 = _____ | (j) 423 x 37 = _____ | (q) 953 x 62 = _____ |
| (d) 235 x 24 = _____ | (k) 175 x 63 = _____ | (r) 188 x 85 = _____ |
| (e) 293 x 53 = _____ | (l) 835 x 24 = _____ | (s) 370 x 60 = _____ |
| (f) 323 x 64 = _____ | (m) 933 x 86 = _____ | (t) 687 x 56 = _____ |
| (g) 537 x 75 = _____ | (n) 732 x 55 = _____ | (u) 893 x 73 = _____ |



Lesson A4.6 Multiplying Large Numbers

We may multiply numbers by arranging their digits in columns by their place values.

1. The same procedure is used when multiplying larger numbers.

Multiply 547 by 213

- (a) There are as many partial products as there are digits in the second factor (multiplier).
 (b) Write the partial products with trailing zeros omitted.
 (c) Start writing the partial products, from right to left, starting from the column of the digit of the second factor.

$$\begin{array}{r}
 \\
 \\
 \\
 \hline
 1 \\
 5 \\
 1 \\
 \hline
 1
 \end{array}$$

Do this multiplication on paper and then check out the product **116,511** on a calculator.

Multiply 526 by 602

- (a) When there is a '0' digit in the second factor, its partial product is zero.
 (b) The row for the partial product of zero may be omitted.
 (c) Keep in mind that the row for the next partial product should line up with the column of the digit in the second factor.

$$\begin{array}{r}
 \\
 \\
 \\
 \hline
 3 \\
 \\
 1 \\
 \hline
 3
 \end{array}$$

Do this multiplication on paper and then check out the product **316,652** on a calculator.

2. We may multiply in two different ways with the same factors to verify the product. The product should be the same in both cases as shown below.

Multiply 21 by 3005

$$\begin{array}{r}
 \\
 \\
 \\
 \hline
 6 \\
 \\
 \hline
 6
 \end{array}$$

$$\begin{array}{r}
 \\
 \\
 \\
 \hline
 3 \\
 \\
 \\
 \hline
 6
 \end{array}$$

☺ Exercise A4.6

1. Multiply the following numbers.

- | | | |
|---------------|-----------------|-----------------|
| (a) 325 × 221 | (f) 1700 × 400 | (k) 121 × 989 |
| (b) 496 × 143 | (g) 2370 × 2000 | (l) 8530 × 778 |
| (c) 333 × 333 | (h) 3059 × 206 | (m) 249 × 117 |
| (d) 574 × 203 | (i) 4444 × 2222 | (n) 3000 × 3000 |
| (e) 408 × 307 | (j) 5333 × 1071 | (o) 2200 × 3030 |

2. Multiply the following, and then verify the product by multiplying the factors in a different order.

- | | | |
|--------------|---------------|-----------------|
| (a) 38 × 56 | (c) 934 × 552 | (e) 3064 × 205 |
| (b) 487 × 75 | (d) 2657 × 35 | (f) 4007 × 1706 |

Answer: 1. (a) 71825 (b) 70928 (c) 110889 (d) 116522 (e) 125256 (f) 680000 (g) 4740000
 (h) 630154 (i) 9874568 (j) 5711643 (k) 119669 (l) 6636340 (m) 29133 (n) 90000000
 2. (a) 2128 (b) 36525 (c) 515568 (d) 92995 (e) 628120 (f) 6835942

Lesson A4.7 Word Problems

To solve a word problem one must translate it from English into a mathematical expression.

1. Some of the verbal expressions that may be translated as addition are as follows. *NOTE: The letter 'n' means a number.*

The number multiplied by 5	5n
10 times the number.....	10n
The product of 23 and the number.....	23n
Twice (or double) the number.....	2n
Triple (or treble) the number.....	3n
Quadruple the number	4n
Quintuple the number	5n

2. To solve a word problem one must be able to think logically with all the words used in describing that problem.

- (a) What is the cost of 93 printers, if each printer costs \$75?

SOLUTION:

$$\begin{aligned}
 \text{The cost of one printer} &= \$75 \\
 \text{Therefore, the cost of 75 printers} &= \$75 \times 93 \\
 &= \$6975 \qquad \qquad \qquad \text{Answer}
 \end{aligned}$$

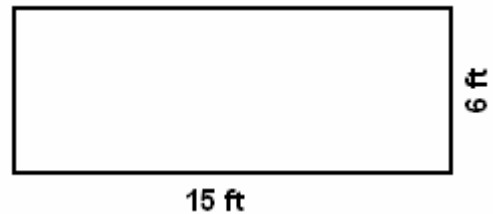
- (b) If 829 men could dig a canal bed in 248 days, how many men will be required to complete the work in 1 day?

SOLUTION:

$$\begin{aligned}
 \text{More man will be required to complete the work in less number of days.} \\
 \text{Therefore, the number of men required to complete the work in 1 day} \\
 &= 829 \times 248 \\
 &= 205,592 \text{ men} \qquad \qquad \qquad \text{Answer}
 \end{aligned}$$

- (c) The figure on the right is called a rectangle. What is the area of the rectangle shown?

*NOTE: The **area** of a figure is how much surface is enclosed within its lines. For example, floor area of a room would be the surface of the floor enclosed within the walls.*



SOLUTION:

A square tile of "1 foot x 1 foot" shall represent a unit area of "1 square foot." The number of such tiles that fit within a surface would then provide the area of that surface.

In the rectangle above, we can imagine 15 “tiles” to fit from left to right, and 6 “tiles” to fit from bottom to top. We may cover the whole rectangle with 6 rows of 15 “tiles” each.

$$\begin{aligned} \text{Number of "tiles" required to cover the whole rectangle} &= 15 \times 6 = 90 \\ \text{Therefore, area of the rectangle} &= 90 \text{ square feet} \qquad \text{Answer} \end{aligned}$$

- (d) Find the area of a rectangular box that is 11 inch long and 8 inch wide.

SOLUTION:

In this case we may use a **square tile** of “1 inch x 1 inch” to represent a **unit area** of “1 square inch.”

$$\begin{aligned} \text{Number of "tiles" required to cover the length} &= 11 \\ \text{Number of "tiles" required to cover the width} &= 8 \\ \text{Therefore, area of the rectangle} &= 11 \times 8 = 88 \text{ square inch} \qquad \text{Answer} \end{aligned}$$



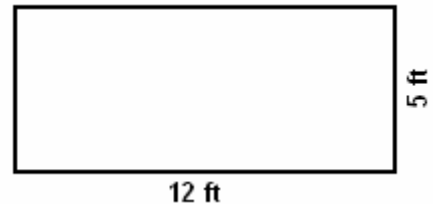
From the last two examples we get the following formula,

$$\text{Area of rectangle} = \text{Length} \times \text{Width}$$

☺ Exercise A4.7

- What is the cost of 65 printers, if each printer costs \$75?
- What will be the weight of 96 horses, estimating them at 1387 pounds apiece?
- A company of soldiers contains 85 men. How many men are there in 67 companies?
- A ream of paper contains 500 sheets. A school uses 1,367 reams in a year. How many sheets of paper were used in that time?
- If a man’s car cost him 12¢ per mile to operate, how much does 30,000 miles of driving cost him in \$?
- If 4 men can paint a house in 3 days, how many men will it take to paint the house in 1 day?
- If 7 people can paint a house in 3 days, how many men will it take to paint the house in 1 day?
- Find the area (number of square feet) in the rectangle shown below on the right.
- A room is 5 yards wide and 7 yards long.
 - Find the floor area of the room.
 - Find the perimeter of the room.

NOTE: the opposite sides of a rectangle are equal.
- A room is 8 yards wide and 9 yards long.
 - Find the floor area of the room.
 - Find the perimeter of the room.
- A rectangular window is 7 feet wide and 4 feet high. How many square feet of glass are needed for this window?
- What does it cost to replace a large bathroom mirror that measures 7 feet by 3 feet if glass costs \$2 per square foot?



- (m) Jan needs drapes that are 3 yards high and 5 yards long to cover a picture window in her home. If she pays \$7 a square yard for the material she likes, what will the drapes cost?
- (n) The floor of a small balcony is 6 feet wide and 9 feet long. How many square feet of material are needed to cover this floor?
- (o) An open box is 8 inches high, 9 inches long, and 6 inches wide. Find the total area of the four sides of the box. *Hint: Each side of the box is a rectangle with an area you know how to find.*
- (p) A rectangular room is 6 yards long and 5 yards wide.
 (i) Find the floor area of this room.
 (ii) Find the cost to carpet this room if the carpet costs \$8 per square yard to install.
- (q) A merchant bought one box of goods for \$250, two more for \$325 each, and three more for \$175 each. He sold them all so as to gain \$356. For how much did he sell them?
- (r) A farmer bought 24 sheep, at \$5 a head; 36 hogs at \$14 a head; and 9 cows at \$45 a head. When he sold them all he lost \$276. For how much did he sell them?

Answer: (a) \$4875 (b) 133,152 pounds (c) 5695 men (d) 683,500 sheets (e) \$3,600 (f) 12 men (g) 21 men (h) 60 sq. feet (i) area = 35 sq. yards, perimeter = 24 yards (j) area = 72 sq. yards, perimeter = 34 yards (k) 28 sq. feet (l) \$42 (m) \$105 (n) 54 sq. feet (o) 240 sq. inch (p) area = 30 sq. yards, \$240 (q) \$1781 (r) \$753

SUMMARY

Multiplication is “repeated addition.” Knowing the numbering system, and the techniques of addition, one can learn to multiply easily.

The proper approach to learning the multiplication table is to create the multiplication table as many times as necessary. You create the table by counting by adding a number to itself repeatedly.

In mixed operations, multiplication takes a priority over addition and subtraction.

The real shortcuts in math come from thinking with the basics, such as, the **Distributive Property**.

DIAGNOSTIC TEST

1. Tell without adding which of the following is equal and why?
(e) $2 + 2 + 2 = 3 + 3$
(f) $5 + 5 + 5 = 3 + 3 + 3 + 3 + 3$
(g) $7 + 7 + 7 = 3 + 3 + 3 + 3 + 3 + 3$
(h) $9 + 9 + 9 = 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3$
2. Multiply the following mentally.
(d) 2×9 (d) 8×3 (g) 9×5
(e) 5×8 (e) 4×7 (h) 8×6
(f) 9×6 (f) 7×9 (i) 7×6
3. Compute the following mentally.
(g) 375×0 (g) 32×5 (m) 70×40
(h) 375×1 (h) 17×5 (n) 7×13
(i) 125×10 (i) 18×2 (o) 8×37
(j) 16×100 (j) 14×4 (p) 6×35
(k) 24×1000 (k) 13×8 (q) 30×7
(l) 6×80 (l) 45×9 (r) 45×99
4. Multiply the following on paper by columns.
(c) 43×2 (c) 123×13 (e) 507×203
(d) 52×3 (d) 175×63 (f) 3005×3005
5. Solve the following word problems.
(a) What is the cost of 65 printers, if each printer costs \$75?
(b) If 4 men can paint a house in 3 days, how many men will it take to paint the house in 1 day?
(c) Jan needs drapes that are 3 yards high and 5 yards long to cover a picture window in her home. If she pays \$7 a square yard for the material she likes, what will the drapes cost?
(d) A farmer bought 24 sheep, at \$5 a head; 36 hogs at \$14 a head; and 9 cows at \$45 a head. When he sold them all he lost \$276. For how much did he sell them?

Answer: 1. (a) 2×3 and 3×2 are equal (b) 5×3 and 3×5 are equal (c) 7×3 and 3×7 are not equal (d) 9×3 and 3×9 are equal 2. (a) 18 (b) 40 (c) 54 (d) 24 (e) 28 (f) 53 (g) 45 (h) 48 (i) 42 (j) 160 (k) 104 (l) 1599 3. (a) 0 (b) 375 (c) 1250 (d) 1500 (e) 24000 (f) 480 (g) 160 (h) 85 (i) 36 (j) 56 (k) 104 (l) 405 (m) 2800 (n) 91 (o) 296 (p) 210 (q) 210 (r) 4455 4. (a) 86 (b) 156 (c) 1599 (d) 11025 (e) 102921 (f) 9030025 5. (a) \$4875 (b) 12 men (c) \$105 (d) \$753

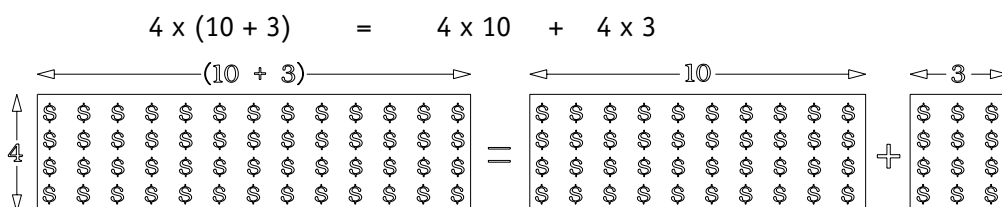
GLOSSARY

[For additional words refer to the glossaries at the end of earlier Milestones]

Associative To **associate** is to accompany together. Therefore, **associative** describes the condition of accompanying or joining together. Multiplication is associative, i.e. when three numbers are to be multiplied; they may associated differently without changing the outcome.

Carry Over A **carry over** is
 (a) Replacing "ten ONES" by "one TEN"
 (b) Replacing "ten TENS" by "one HUNDRED"
 (c) Replacing "ten HUNDREDS" by "one THOUSAND," and so on.

Distributive Multiplication (x) is **distributive** over addition (+). For example,



Commutative To **commute** is to change the order or arrangement of. Therefore, **commutative** describes interchange or exchange. Multiplication is commutative, i.e. two numbers may be multiplied in any order without changing the outcome.

Multiplicand A **multiplicand** is the number, which is being multiplied. A multiplicand is also called a factor of the product.

Mixed Operation In **mixed operations**, multiplication is carried out before addition because the operation of multiplication is of a higher order than addition.

Multiplication When a number is added to itself repetitively we call it **multiplication**. Since several additions are compressed to make one multiplication, the operation of multiplication is considered of a higher order than simple addition.

Multiplication Sign The sign "x" when placed between two numbers, expresses that those two numbers are to be multiplied.

Multiplier A **multiplier** is the number of times the multiplicand is multiplied. A multiplier is also called a factor of the product.

Parentheses **Parentheses** are used to show that all included terms are to be considered together as a single term.

Product A **product** is the result of multiplication.

Turn about **Turn about** means to switch around the roles of coach and student while doing an exercise.