MATH MILESTONE # A2

ADDITION

The word, **milestone,** means "a point at which a significant change occurs." A Math Milestone refers to a significant point in the understanding of mathematics.

To reach this milestone one should be able to add two single- and double-digit numbers mentally without effort.

A simple ABACUS, as described in Milestone #1, shall be required. Alternately, you may access the **VIRTUAL ABACUS** at *www.mathfundamentals.org/abacus.htm* for the purpose of this milestone. Also have some pennies, paper and pencil, and a calculator at hand.

Please consult the **Glossary** supplied with this Milestone for mathematical terms. Consult a regular dictionary at *www.dictionary.com* for general English words that one does not understand fully.

You may start with the Diagnostic Test on the next page to assess your proficiency on this milestone. Then continue with the lessons with special attention to those, which address the weak areas.

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DIAGNOSTIC TEST

1. Count the following together.
   (a) 7 horses and 5 horses
   (b) 7 horses and 5 camels
   (c) 6 hours and 3 hours
   (d) 5 hours and 30 minutes
   (e) 2 feet and 3 inches
   (f) 2 inches and 3 inches

2. Solve the following problems.
   (a) For his birthday, Peter got 3 books from his uncle, and 3 books from his aunt. How many books did Peter receive on his birthday?
   (b) You caught 2 butterflies on Monday and 5 butterflies on Tuesday. How many butterflies did you catch in all?
   (c) Elli had 4 dolls. She got 5 more on her birthday. How many dolls does Elli have?

3. Add the following numbers by <u>counting more</u> on your fingers.
   (a) 9 + 3
   (b) 3 + 19
   (c) 9 + 20
   (d) 5 + 23
   (e) 87 + 7
   (f) 9 + 99
   (g) 8 + 37
   (h) 14 + 5
   (i) 6 + 86

4. Add the following on abacus.
   (a) 3 + 6
   (b) 9 + 3
   (c) 8 + 6
   (d) 24 + 6
   (e) 44 + 3
   (f) 56 + 7
   (g) 52 + 27
   (h) 79 + 21
   (i) 88 + 65

5. Add the following mentally.
   (a) 10 + 9
   (b) 20 + 5
   (c) 44 + 10
   (d) 93 + 10
   (e) 64 + 9
   (f) 53 + 8
   (g) 66 + 7
   (h) 48 + 5
   (i) 79 + 6
   (j) 45 + 38
   (k) 79 + 14
   (l) 97 + 45

6. Count up to 20 by 2. (NOTE: This is counting 2, 4, 6... to 20.)

7. Count from 7 to 70 by 7.

8. Add the following mentally. Check your answer by adding in reverse order.
   (a) 2 + 3 + 8 + 7 + 6 + 4 + 8 + 9   = _____
   (b) 7 + 9 + 3 + 8 + 1 + 8 + 9 + 2   = _____
   (c) 3 + 6 + 6 + 4 + 1 + 2 + 7 + 8   = _____

9. Give the first ten counts by 12.

10. Give the first ten counts by 17.

11. Add the following by using columns on paper and pencil.
    (a) 463 + 359 + 706 = _____
    (b) 3,078 + 5,972 + 6,876 = _____
    (c) 23,087 + 15,223 + 309,854 = _____

Answer: 1. (a) 12 horses (b) 12 camels (c) 9 hours (d) Cannot (e) Cannot (f) 5 inches 2. (a) 6 (b) 7 (c) 9 3. (a) 12 (b) 22 (c) 29 (d) 28 (e) 94 (f) 108 (g) 45 (h) 19 (i) 92 4. (a) 9 (b) 12 (c) 14 (d) 30 (e) 47 (f) 63 (g) 79 (h) 100 (i) 153 5. (a) 19 (b) 25 (c) 54 (d) 103 (e) 73 (f) 61 (g) 73 (h) 53 (i) 85 (j) 83 (k) 93 (l) 142 6. 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 7. 7, 14, 21, 28, 35, 42, 49, 56, 63, 70 8. (a) 47 (b) 47 (c) 37 9. 12, 24, 36, 48, 60, 72, 84, 96, 108, 120 10. 17, 34, 51, 68, 85, 102, 119, 136, 153, 170 11. (a) 1528 (b) 15,926 (c) 348,164

(01/30/2008)

LESSONS

Lesson A2.1   Counting to Addition

ADDITION is counting quantities of the same unit together.

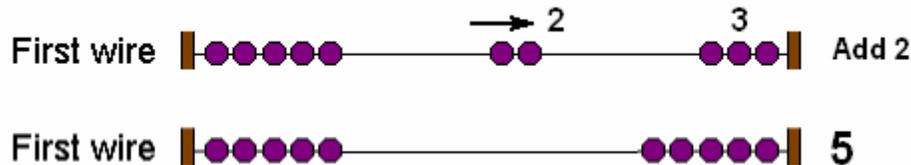1. When we count quantities together we get a total or a SUM.

    (a) When we count 2 fingers and 3 fingers together, we get a total of 5 fingers.

    (b) When we count 2 pennies and 3 pennies together, we get 5 pennies.

    (c) When we count 3 and then 2 more by moving beads on abacus we get a sum of 5.

2. We may add quantities of the same unit only. To add quantities of different units, we must first express them in a common unit.

    (a) We may count 3 cats and 2 cats together as 5 cats. But we cannot count 3 cats and 2 dogs together as a single number. But if we express them as 3 animals and 2 animals, then we may add them as 5 animals.

    (b) Similarly, we may count 3 pennies and 2 pennies together as 5 pennies. But we cannot count 3 pennies and 2 dimes together as a single number. But if we express them as 3 cents and 20 cents before, then we may add them as 23 cents.

    (c) Whenever we add two numbers on abacus, we assume that they have the same units.

3. In general, when we add 2 counts and 3 counts of anything we get 5 counts of that thing. This makes it possible to count other things using fingers, abacus, or just mentally.

☺ Exercise A2.1

1. Count the following together to get a total.
   (a) 5 marbles and 2 marbles
   (b) 6 quarters and 3 quarters
   (c) 2 feet and 4 feet

2. Add the following to get a sum.
   (a) 3 horses and 5 camels
   (b) 3 horses and 5 horses
   (c) 5 horses, 4 camels, 2 horses and 3 camels

3. Add the following using a common unit?
   (a) 4 apples and 3 bananas
   (b) 5 men and 5 women
   (c) 3 $10-bills and 5 $1-bills
   (d) 4 TENS, 3 ONES and 1 TEN

4. If you don't have cats how you would you know the sum of 4 cats and 5 cats?

Lesson A2.2   Basics of Addition

*Adding is counting more from a number.*

1. To add 5 and 3, start from 5 and count 3 more (6, 7, 8). The sum is 8.

2. We write addition with a plus sign (+) between the numbers to be added. For example, if we are adding the counts 5 and 3, we may express it as follows.

$$5 \; + \; 3 \quad = \quad 8$$

   (a) "**+**" is the **plus sign** for addition; "**=**" is the **equals sign**.
   (b) We read the above expression as, "**5 plus 3 equals 8.**"
   (c) The **units** of all numbers above are assumed to be the same.
   (d) The numbers to be added, 5 and 3, are called **addends** (to be added).
   (e) The outcome of addition, 8, is called the **sum.**

3. We may add two numbers in any order. For example, we may count 3 more from 5, or count 5 more from 3. The sum is the same (8) in either case.

$$5 + 3 \quad = \quad 3 + 5$$

   (a) This is called the **Commutative Property of Addition**. The word COMMUTE means, "To change the order or arrangement of."

   (b) Therefore, to add, we may start with the larger number and count as many more as the smaller number.

4. For sums less than ten, we simply count the number together as shown above.

   (a) To add either "3 and 5," or "5 and 3," we start from 5 and count 3 more. The sum is 8.
   Note: It is okay to use fingers to count for sums less than ten.

   (b) To add 3 and 5, we count 5 and then 3 more, to get the sum of 8 on the right.
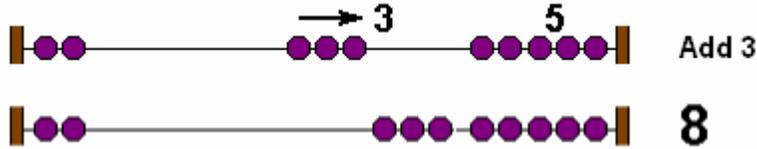
   (c) Only the first wire of abacus is needed to count sums less than ten. You may access a virtual abacus at *www.mathfundamentals.org/abacus.htm*

5. Some simple word problems require addition only:

   (a) John ate 5 candies. He then ate 2 more. How many candies did he eat in all?
   This is clearly an addition problem. We start from 5 and count 2 more, getting a sum of 7. Therefore, John ate 7 candies in all.

   (b) Bill has 6 cards. He traded 4 more for some of his stamps. How many cards does he now have?
   This is also an addition problem. We add 6 and 4. The sum is 10. Therefore, Bill has 10 cards now.

☺ Exercise A2.2

1. Write down "3 and 2 equal 5" using mathematical symbols.

2. Identify the addends and sum in the following.
   (a) 9 + 3 = 12   (b) 5 + 23 = 28   (c) 8 + 37 = 45

3. State the order in which you will add 3 and 16 mentally. Why?

4. Add the following by starting from the larger addend.
   (a) 7 + 2   (d) 3 + 4   (g) 8 + 1
   (b) 1 + 7   (e) 6 + 3   (h) 2 + 6
   (c) 4 + 5   (f) 5 + 2   (i) 5 + 4

5. Practice adding numbers from the table on the right. *Note: For the addends in yellow row and column respectively, the sum is in a white square where the respective column and row from those numbers meet.*

   Example: The sum of 4 and 3 is 7 as shown.

   Have another person call out addends from the yellow squares, for which the sum is available on a white square. Count the sum using fingers, abacus, or mentally.

   If possible, do this exercise on a turn about basis, Turn about means," To switch around the roles of coach and student while doing an exercise."

Sums less than Ten

	0	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9		
2	3	4	5	6	7	8	9			
3	4	5	6	7	8	9				
4	5	6	7	8	9					
5	6	7	8	9						
6	7	8	9							
7	8	9								
8	9									
9										

(01/30/2008)

6. Count and verify the sum for the numbers at the following link
   www.themathpage.com/ARITH/elementary.htm#less

7. Solve the following using addition.

   (a) For his birthday, Peter got 3 books from his uncle, and 3 books from his aunt. How many books did Peter receive on his birthday?

   (b) You caught 2 butterflies on Monday and 5 butterflies on Tuesday. How many butterflies did you catch in all?

   (c) Elli had 4 dolls. She got 5 more on her birthday. How many dolls does Elli have?

Answer: 1. 3 + 2 = 5   2. Addends are on the left of the "=" sign, and sum is on the right. 3. Start with the larger number 16, and count 3 more to get the sum 19. 4. (a) 9 (b) 8 (c) 9 (d) 7 (e) 9 (f) 7 (g) 8 (h) 8 (i) 9 7. (a) 6 (b) 7 (c) 9
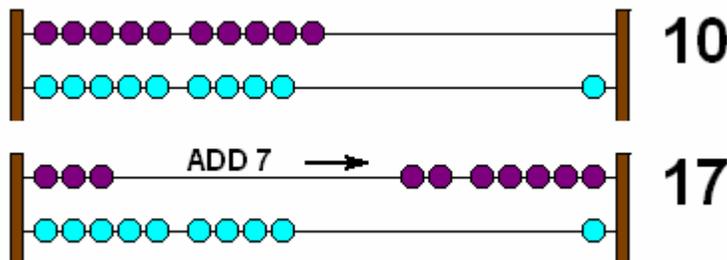
Lesson A2.3   Sums between 10 than 20

We may add larger sums by counting more; however adding to TEN speeds up the process.

1. An advantage of TEN is that you can add another number to it easily.

   (a) To add a single-digit number to 10, we simply replace the '0' in 10 by that single-digit number.
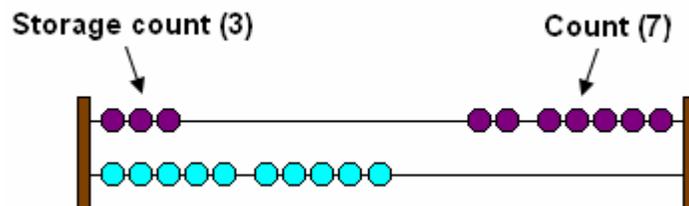
   $$10 + 7 \quad = \quad 17$$

   This may be visualized on an abacus as follows.

   This gives us the following rule, where **N** is a single-digit number.
 $$10 + N \quad = \quad \text{'1N'} \qquad \text{(Replace '0' by N)}$$

2. On a wire of abacus, the counted beads are moved to the right, and the uncounted beads remain in "storage" on the left. The total of counted and uncounted beads is always 10.

   (a) If "storage count" is the number of uncounted beads, then we may say that the sum of a "count" and its "storage count" is ten. We may define the "storage count" is that additional count that takes a single-digit number to ten.

(b) We may demonstrate the function of "storage count" by adding **8** and **5** on abacus:

    (1) We count 8 beads to the right on the first wire. *See figure on the next page.*

    (2) To count 5 more, we first count the 2 remaining beads on the first wire. *Note: This is the "storage count."*

    (3) No more beads remain on the first wire to count. So, we regroup the 10 beads on the first wire as 1 bead on the second wire.

    (4) Now we have more beads to count on the first wire. So, we count the remaining 3 beads.

    (5) We read the sum on the right as 13.
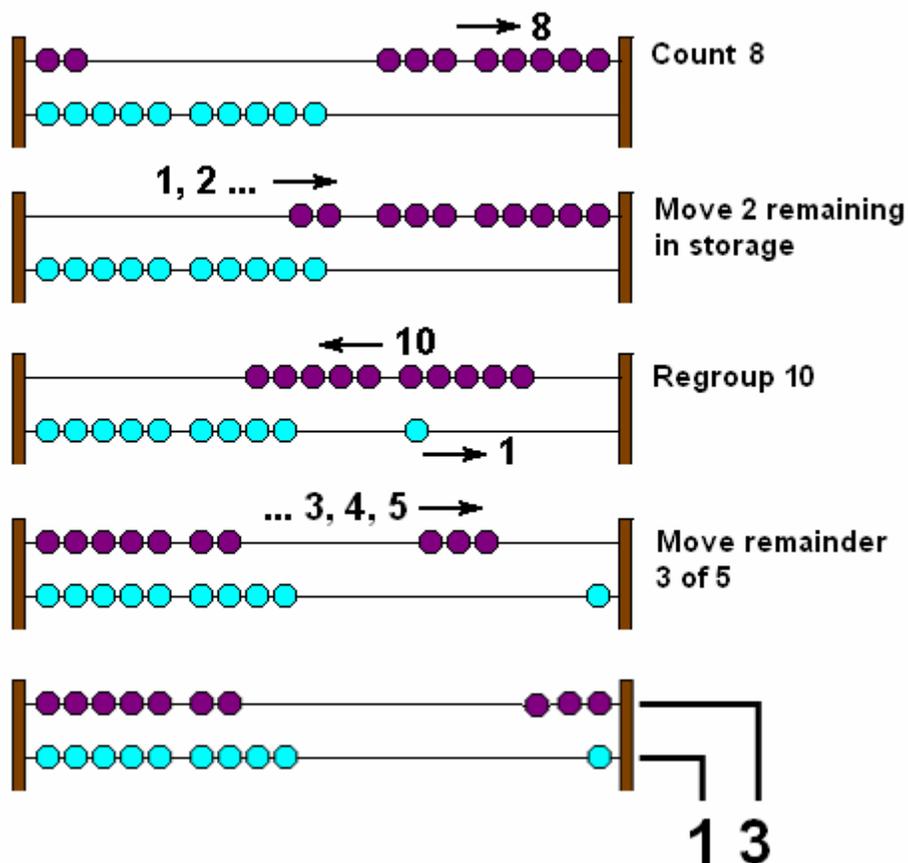
(c) We may add 8 and 5 mentally as follows.

    We "borrow" the storage count for 8 (which is 2) from 5. The first addend becomes 10, and the second addend becomes 3. We add 10 and 3 to get the sun of 13.

$$8 + 5 \ = \ 8 + (2 + 3) = \ (8 + 2) + 3 = \ 10 + 3 \ = \ 13$$

(d) We may add 9 and 6 as follows.

$$9 + 6 \ = \ 9 + (1 + 5) = \ (9 + 1) + 5 = \ 10 + 5 \ = \ 15$$

(e) As demonstrated above, we may group three numbers in two different ways, without changing their sequence. The outcome is the same.

$$9 + (1 + 5) \quad = \quad 15$$
$$(9 + 1) + 5 \quad = \quad 15$$

This is called the **Associative Property of Addition**. The meaning of the word ASSOCIATE is, "to group together."

☺ Exercise A2.3

1. Add 10 and a single-digit number quickly.
   (a) 10 + 2   (c) 9 + 10   (e) 10 + 6   (g) 8 + 10   (i) 10 + 5
   (b) 4 + 10   (d) 10 + 7   (f) 1 + 10   (h) 10 + 3   (j) 0 + 10

2. Have another person call out a single-digit number randomly. Add 10 to that number quickly. Repeat this exercise until you can do so effortlessly. *Note: If possible, do this exercise on a turn about basis.*

3. Find the "storage count" for the following numbers.
   (a) 2   (c) 9   (e) 6   (g) 8   (i) 5
   (b) 4   (d) 7   (f) 1   (h) 3   (j) 0

4. Practice finding "storage count" from abacus or from the table below on the left.

   Example: The "storage count" for 7 is 3, and for 3 it is 7.

   Have another person call out single-digit numbers randomly. Find the "storage count" necessary to take that number to TEN. Repeat this exercise until you can do so effortlessly. *Note: If possible, do this exercise on a turn about basis.*

5. Add the following numbers rapidly.
   (a) 9 + 2   (d) 8 + 3   (g) 7 + 5   (j) 6 + 5
   (b) 9 + 3   (e) 8 + 4   (h) 7 + 6   (k) 8 + 6
   (c) 9 + 4   (f) 8 + 5   (i) 7 + 8   (l) 9 + 7

6. Practice adding two single-digit numbers from the table above on the right. *Note: For the addends in yellow <u>row and column</u> respectively, the sum is in a white square where the respective <u>column and row</u> from those numbers meet.*

   Example: The sum of 6 and 5 is 11 as shown.

Have another person call out two single-digit numbers randomly. Add those numbers using fingers, abacus, or mentally using the techniques of this section. Repeat this exercise until you can do so effortlessly. *If possible, do this exercise on a turn about basis.*

7. Count and verify the sum for the numbers at the following link
    www.themathpage.com/ARITH/elementary.htm#prob

8. Practice word problems from the following link as needed. Check your answers on a calculator.
    www.mathfundamentals.org/Exercise/02%201%20Addition.pdf
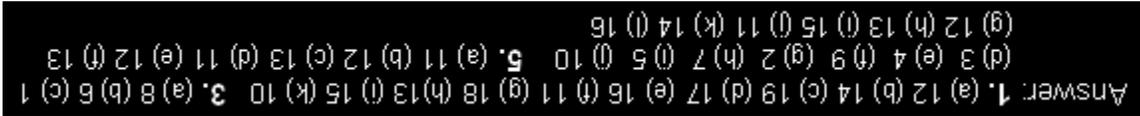
Lesson A2.4   Adding a Single-Digit to a Double-Digit Number

We may add by counting more as earlier. However, we may speed up addition by finding the last digit of the sum first, and then finding the next number, which ends with that digit.

1. The last digit of the sum depends on the last digits of the addends.

    (a) When the last digits of the addends are 9 and 6, the last digit of the sum is always 5.

    $$9 + 6 = 15 \qquad 29 + 6 = 35$$
    $$19 + 6 = 25 \qquad 39 + 6 = 45$$

    (b) When the last digits of the addends are 8 and 5, the last digit of the sum is always 3.

    $$8 + 5 = 13 \qquad 48 + 5 = 53$$
    $$28 + 5 = 33 \qquad 68 + 5 = 73$$

    (c) When the last digits of the addends are 5 and 4, the last digit of the sum is always 9.

    $$5 + 4 = 9 \qquad 55 + 4 = 59$$
    $$35 + 4 = 39 \qquad 75 + 4 = 79$$

2. When we add a single-digit number to a double-digit number, the sum is always the next number, which ends with the predicted digit.

    (a) Add 63 and 4
        We predict the last digit of the sum to be **7** (3 + 4 = 7). Starting from the larger addend, the next number, which ends in 7 is 67. Therefore,
        $$63 + 4 = 67$$

    (b) Add 79 and 6
        We predict the last digit of the sum to be **5** (9 + 6 = 15). Starting from the larger addend, the next number, which ends in 5 is 85. Therefore,
        $$79 + 6 = 85$$

    (c) Add 7 and 45
        We predict the last digit of the sum to be **2** (7 + 5 = 12). Starting from the larger addend, the next number, which ends in 2 is 52. Therefore,
        $$7 + 45 = 52$$

☺ **Exercise A2.4**

1. Tell the last digit of the sums.
   (a) 33 + 8   (c) 78 + 6   (e) 72 + 5   (g) 45 + 8   (i) 44 + 5
   (b) 58 + 7   (d) 68 + 8   (f) 26 + 6   (h) 59 + 9   (j) 63 + 5

2. Have another person call out double- and single-digit numbers randomly. Provide only the last digit of the sum. Repeat this exercise until you can do so effortlessly. *If possible, do this exercise on a turn about basis.*

3. Add the following rapidly by getting the last digit of sum first.
   (a) 53 + 8   (c) 48 + 6   (e) 82 + 5   (g) 65 + 8   (i) 94 + 5
   (b) 38 + 7   (d) 28 + 8   (f) 66 + 6   (h) 79 + 9   (j) 33 + 4

4. Add the following rapidly by any technique.
   (a) 9 + 4   (e) 79 + 3   (i) 58 + 7   (m) 68 + 7   (q) 79 + 6
   (b) 19 + 4   (f) 38 + 6   (j) 57 + 5   (n) 77 + 8   (r) 88 + 5
   (c) 28 + 6   (g) 39 + 5   (k) 48 + 4   (o) 89 + 5   (s) 83 + 7
   (d) 37 + 6   (h) 47 + 4   (l) 59 + 7   (p) 67 + 7   (t) 98 + 6

5. Have another person call out double- and single-digit numbers randomly. Add those numbers using fingers, abacus, or mentally using the techniques of this section. Repeat this exercise until you can do so effortlessly. *If possible, do this exercise on a turn about basis.*

Lesson A2.5   Adding Double-Digit Numbers

We add TENS to TENS, and ONES to ONES.

1. On abacus, we first add the TENS and then the ONES.

   (a) Add **15** and **23:**
       (1) Count 15 for the first addend as 1 TEN and 5 ONES.
       (2) Count 2 TENS and then 3 ONES for the second addend.
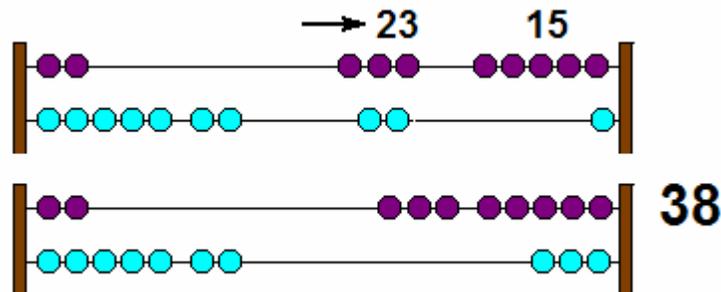
       (3) The sum may be read on the right as 38.

   $$23 + 15 \quad = \quad 38$$

   (b) Add **37** and **25**
       (1) Count 37 for the first addend as 3 TENS and 7 ONES.
       (2) Count 2 TENS for the second addend.

(3) Count 5 ONES for the second addend as follows.
- ➢ Count the "storage" of 3 ONES
- ➢ Regroup 10 ONES to 1 TEN.
- ➢ Count remaining 2 ONES.

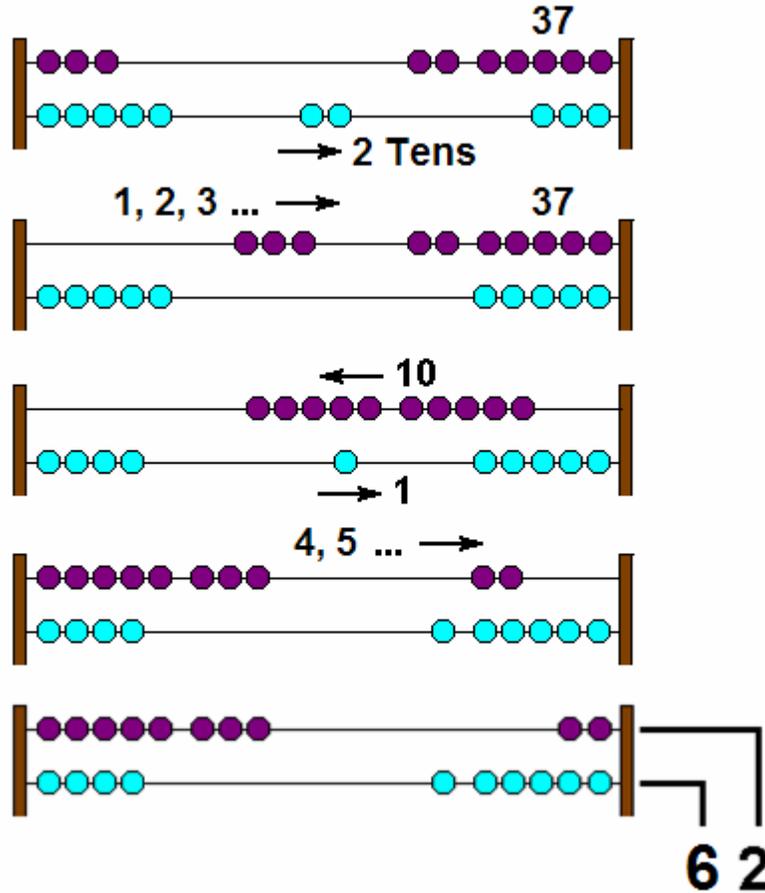(4) The sum may be read on the right as 62.

$$37 + 25 \quad = \quad 62$$

(c) Mentally also, we may add the TENS first and then the ONES

49 + 26	→	49 + 20 + 6	→	69 + 6	→	75
33 + 28	→	33 + 20 + 8	→	53 + 8	→	61
57 + 35	→	57 + 30 + 5	→	87 + 5	→	92

2. We are always free to use other ways of adding.

(a) We may first add just the TENS together, and then add just the ONES together. And then we add the resulting sums together.

49 + 26	→	60 + 15	→	70 + 5	→	75
33 + 28	→	50 + 11	→	60 + 1	→	61
57 + 35	→	80 + 12	→	90 + 2	→	92

(b) We may also add two numbers that are close to each other, by first doubling the smaller number and then adding the difference.

6 + 7	=	6 + 6 + 1	=	12 + 1	=	13
4 + 5	=	4 + 4 + 1	=	8 + 1	=	9

16 + 17	=	Double 16 and add 1	=	32 + 1	=	33
44 + 46	=	Double 44 and add 2	=	88 + 2	=	90
35 + 38	=	Double 35 and add 3	=	70 + 3	=	73

☺ Exercise A2.5

1. Add the following double-digit numbers using the techniques of this section. Verify your answers on a calculator.

   (a) 33 + 28   (g) 45 + 38   (m) 36 + 28
   (b) 58 + 37   (h) 51 + 74   (n) 48 + 27
   (c) 77 + 44   (i) 47 + 86   (o) 89 + 11
   (d) 64 + 27   (j) 28 + 23   (p) 68 + 38
   (e) 72 + 25   (k) 57 + 37   (q) 97 + 45
   (f) 26 + 46   (l) 79 + 14   (r) 36 + 26

2. *Have another person call out two double-digit numbers randomly. Add those numbers using fingers, abacus, or mentally using the techniques of this section. Verify the sum on calculator to build up confidence. Repeat this exercise until you can do so effortlessly. If possible, do this exercise on a turn about basis.*

3. Double the following numbers using earlier methods.
   (a) 5   (b) 9   (c) 6   (d) 3   (e) 7   (f) 12   (g) 17   (h) 25

4. Have another person call out numbers less than 100 randomly. Double the number mentally. Repeat this exercise until you can do so effortlessly. *If possible, do this exercise on a turn about basis.*

5. Add by doubling the smaller number and then adding 1.
   (a) 5 + 6   (c) 8 + 9   (e) 6 + 7
   (b) 7 + 8   (d) 4 + 5   (f) 3 + 4

6. Add the following by doubling the first number and then adding the difference.
   (a) 15 + 16   (d) 64 + 65   (g) 50 + 53
   (b) 12 + 13   (e) 36 + 37   (h) 48 + 49
   (c) 18 + 19   (f) 41 + 43   (i) 75 + 77

7. *Have another person call out two double-digit numbers close to each other. Add them mentally using the doubling technique. Verify the sum on calculator to build up confidence. Repeat this exercise until you can do so effortlessly. If possible, do this exercise on a turn about basis.*

Answer: 1. (a) 61 (b) 95 (c) 121 (d) 91 (e) 97 (f) 72 (g) 83 (h) 125 (i) 133 (j) 51 (k) 94 (l) 93 (m) 64 (n) 75 (o) 100 (p) 106 (q) 142 (r) 62 3. (a) 10 (b) 18 (c) 12 (d) 6 (e) 14 (f) 24 (g) 34 (h) 50 5. (a) 11 (b) 15 (c) 17 (d) 9 (e) 13 (f) 7 6. (a) 31 (b) 25 (c) 37 (d) 129 (e) 73 (f) 84 (g) 103 (h) 97 (i) 152

☺ Practice #1

1. Add the following from left to right and then from right to left. See if you get the same answer.

   (a) 2 + 3 + 8 + 7 + 6 + 4 + 8 + 9   (d) 8 + 7 + 5 + 9 + 6 + 7 + 2 + 7
   (b) 7 + 9 + 3 + 8 + 1 + 8 + 9 + 2   (e) 5 + 6 + 3 + 5 + 4 + 8 + 9 + 9
   (c) 3 + 6 + 6 + 4 + 1 + 2 + 7 + 8

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2. Add the following mentally writing down only the answers. Then compare the answers with those given below.

(a) 32 + 11	(e) 77 + 13	(i) 28 + 14	(m) 25 + 23	(q) 48 + 26
(b) 53 + 13	(f) 33 + 17	(j) 37 + 15	(n) 44 + 33	(r) 68 + 25
(c) 28 + 16	(g) 58 + 13	(k) 48 + 14	(o) 35 + 25	
(d) 37 + 22	(h) 65 + 15	(l) 59 + 17	(p) 67 + 27	

3. Count by 2, 3, 4, 5, 6, 7, 8, 9, and 10 using **repeated addition**. For example, to count by 2, you shall count 2, 4, 6, 8, and so on, by repeatedly adding 2. Write down the first ten counts in columns, and then check them against the table below. *If possible, do this exercise on a turn about basis.*

4. Count by beyond 10 using **repeated addition** as you deem necessary for practice. You may check your counts against the table below.

Repeated Addition

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	9	12	15	16	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200

Lesson A2.6   Adding by column

To add larger numbers on paper, first arrange their digits under each other by place values; and then add the columns of digits from right to left. Use the Rule of Abacus as "carry over."

1. The mental steps are for these additions are detailed below

Add 13 and 32
(a) For ONES: 3 plus 2 equals 5 ONES;
(b) For TENS: 1 plus 3 equals 4 TENS.
(c) The sum is 45.

```
      tens  ones
        1    3    addend
  +     3    2    addend
      _____
        4    5    sum
```

Add 53 and 29.
(a) For ONES: 3 plus 9 equals 12; put down 2 ONES and carry over 1 TEN;
(b) For TENS: 1 carried over with 5, plus 2 equals 8 TENS.
(c) The sum is 82.

```
        1         carry-over
        5    3    addend
  +     2    9    addend
      _____
        8    2    sum
```

2. When adding several large numbers, the procedure remains the same, though the carry-overs may be larger:

Add 1637, 286 and 979.
(a) For ONES: 7 plus 6 plus 9 equals 22; put down 2 ONES; carry over 2 TENS;
(b) For TENS: 2 carried over with 3, plus 8 plus 7 equals 20 TENS; put down 0; carry over 2 HUNDREDS;
(c) For HUNDREDS: 2 carried over with 6, plus 2 plus 9 equals 19 HUNDREDS; put down 9; carry over 1 THOUSAND;
(d) For THOUSANDS: 1 carried over with 1 equals 2 THOUSANDS.

```
      1 2 2
      1 6 3 7
        2 8 6
 +      9 7 9
     ─────────
      2 9 0 2
```

3. To check for the correctness of the sum, add the numbers again in a different order.

Add 657, 319 and 825.

```
      6 5 7            8 2 5
      3 1 9            3 1 9
 +    8 2 5       +    6 5 7
   ─────────         ─────────
    1 8 0 1          1 8 0 1
```

☺ Exercise A2.6

1. Add the following numbers. Check your answers on a calculator. (HINT: When digits in the tens column add up to more than nine, carry over to the hundreds column.)

```
   (a)           (b)           (c)           (d)           (e)           (f)

     5 4          7 8           7 5           9 9           4 5           3 3
 +   1 7     +      4       +   2 8       +   3 1       +   3 7       +   4 7
 ────────    ────────      ────────      ────────      ────────      ────────

```

2. Add the following by arranging them in column on paper.
(a) 32 + 16   (d) 47 + 32   (g) 71 + 18
(b) 52 + 23   (e) 54 + 43   (h) 65 + 23
(c) 44 + 33   (f) 36 + 32   (i) 73 + 24

3. Add the following numbers. Check your answers on a calculator.

```
     (a)                (b)                 (c)                     (d)

   2 7 7 3          3 7 9             7 5 9 5 3 3           8 7 2 1 5
   2 7 8            2 2 1             3 6 2 8 1 1           3 5 7 3 2
 + 6 3 4        +   5 7 3         +   1 5 3 8 6 4       +     5 6 3 7
 ──────────       ────────         ──────────────         ──────────


     (e)                (f)                 (g)                     (h)

   3 2 5 4          6 6 7             1 4 9 3 5 6           9 9 9 9 9
   3 7 6            4 1 5             8 0 8 0 1             4 4 4 4 4
 + 9 2 7        +   7 9 3         +   7 4 2 7           +   7 7 7 7
 ──────────       ────────         ──────────────         ──────────

```

	(i)	(j)	(k)	(l)
	9 0 2 5	2 9 2	3 4 7 1 0	3 0 0 0 4
	3 1 0 2	7 3 1	9 1 4 0 2 3	8 8 0 9
+	2 5 3	+ 6 5 5	+ 4 0 3 5 5 2	+ 6 3 0 4 8

4. Add the following by arranging them in column and using carry-over.
   (a) 37 + 24   (d) 47 + 38   (g) 23 + 18
   (b) 77 + 15   (e) 24 + 38   (h) 65 + 27
   (c) 46 + 37   (f) 36 + 59   (i) 43 + 29

5. Add the following using columns
   (a) 245 + 537 + 723   (d) 283 + 336 + 597 + 638
   (b) 3006 + 987 + 268   (e) 2003 + 486 + 3102 + 946
   (c) 5183 + 2975 + 3079   (f) 371 + 578 + 607 + 750 + 432

6. Add the following with addends in at least two different orders to check the correctness of your answer.
   (a) 734 + 569   (d) 66 + 37 + 59
   (b) 3,006 + 987   (e) 627 + 358 + 462
   (c) 5,183 + 2,975   (f) 601 + 523 + 599

Answer: 1. (a) 71 (b) 82 (c) 103 (d) 130 (e) 82 (f) 80 (g) 82 (h) 130 (i) 79 (j) 75 (k) 77 (l) 68
2. (a) 48 (b) 75 (c) 77 (d) 79 (e) 97 (f) 68 3. (a) 3685 (b) 5117 (c) 1276208 (d) 1285624 (e) 4557 (f) 1875
(g) 237584 (h) 152220 (i) 123800 (j) 1678 (k) 1352285 (l) 101861 4. (a) 61 (b) 92
(c) 83 (d) 85 (e) 62 (f) 96 (g) 41 (h) 92 (i) 72 5. (a) 1505 (b) 4261 (c) 11237 (d) 1854
6. (a) 1303 (b) 3993 (c) 8158 (d) 162 (e) 1447 (f) 1723

☺ Practice #2

1. Add the following by columns.
   (a) 734 + 569   (f) 6,060,066 + 387,737
   (b) 3,006 + 987   (g) 73,503,627 + 45,732,358
   (c) 5,183 + 2,975   (h) 600,000,001 + 523,987,999
   (d) 74,111 + 37,555   (i) 5,082,359,777 + 3,193,461,857
   (e) 835,254 + 657,578   (j) 73,321,654,987 + 37,789,123,456

2. Add the following by columns.
   (a) 734 + 569 + 863
   (b) 3,006 + 987 + 333 + 2,050
   (c) 5,183 + 2,975 + 1,582 + 2,974
   (d) 23,725 + 85,306 + 33,592 + 42,829
   (e) 305,183 + 82,975 + 1,582 + 3,903,974 + 80,000,009

3. Fill in the blanks.

   (a)
   ```
     □ 2 2 □
     1 □ □ 1
     ───────
     3 4 8 9
   ```

   (b)
   ```
     1 1 3
     6 □ 4
     1 4 □
     □ 2 6
     ───────
     □ 4 1 0
   ```

(01/30/2008)

4. Call out two or more large numbers to be added. Add them by columns. Verify the answers on a calculator. Practice this until you can add with confidence. *If possible, do this exercise on a turn about basis.*

Lesson A2.7 Comparing numbers

We may compare numbers to find how much more is one number than another number.

1. A comparison shows the unmatched portion between two numbers. This tells you how much more is one number than the other. This is also called the difference between the two numbers.

2. We determine the difference by counting from the smaller to the larger number.

(a) Compare 5 pennies to 2 pennies.
   We start from 2 and may count on fingers **"3, 4, 5"** up to 5.
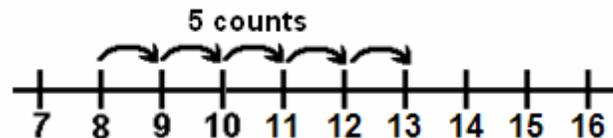   This gives us 3 counts from 2 to 5. This is the unmatched portion. This is the difference.

"The DIFFERENCE is 3 pennies."

   We may also get this answer by asking: **2 + what? = 5**
   Again, this gives us 3 counts as the answer.

(b) You have 8 candies and Bill has 13 candies. How many more candies does Bill have?
   Ask, **8 + what? = 13**

   8 + <u>5</u> = 13. Therefore, 13 is **5** more than 8, or the difference is **5**.
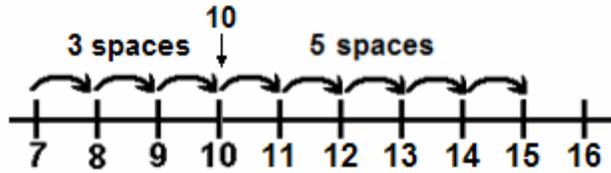
(c) Compare 57 to 63 units.
   Ask, **57 + what? = 63**
   57 + <u>6</u> = 63. Therefore, 63 is **6** more than 57, or the difference is **6**.

3. This difference may also be determined by counting from the smaller number to the nearest TEN, and then from nearest TEN to the larger number.

(a) Find the difference between 7 and 15.
   Ask, **7 + what? = 15**

Count from 7 to 10 and then from 10 to 15   =   3 + 5   =   8

Verify: 7 + <u>8</u> = 15   →   The difference is 8.

(b) Find the difference between 7 and 82.
Ask,   7 + what?   =   82
Count from 7 to 10 and then from 10 to 82   =   3 + 72   =   75

(c) Find the difference between 43 and 26.
Ask,   26 + what?   =   43
Count from 26 to 30 and then from 30 to 43   =   4 + 13   =   17

4. We may use columns to find the difference between large numbers.

(a) Find the difference between 623 and 387.
We solve the same problem by asking: 387 + what? = 623

The mental steps are:
(a) For 1's: "7 + what?" = 13 (the next number ending in 3). The answer is 6. We place 6 under 7 and carry over 1 from 13 to the next column.
(b) For 10's: 8 and 1 makes 9. We ask, "9 + what = 12?" The answer is 3. We place 3 under 8 and carry over 1 from 12 to the next column.
(c) For 100's: 3 and 1 is 4. We ask, "4 + what = 6?" The answer is 2. We place 2 under 3.
(d) The difference is **236**. This is the answer.

(b) Find the difference between 4,001,030,352 and 1,946,327,115.
We solve the same problem by asking
1,946,327,115 + what? = 4,001,030,352
We use the same techniques by column as above.

(c) Find the difference between 5,082,359,777 and 3,193,461,857.
We solve the same problem by asking
3,193,461,857 + what? = 5,082,359,777

☺ Exercise A2.7

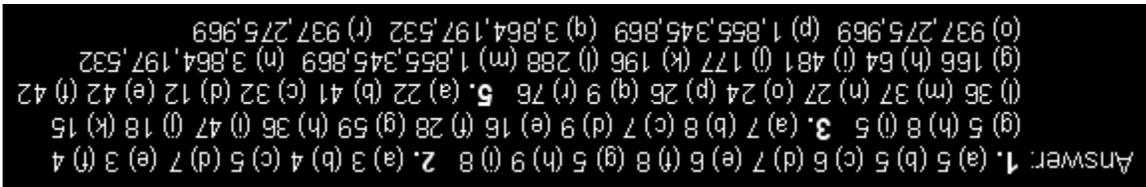1. How much more is the larger number?.
   (a) 4 and 9   (d) 44 and 51   (g) 29 and 34
   (b) 8 and 13   (e) 60 and 66   (h) 58 and 67
   (c) 37 and 43   (f) 82 and 90   (i) 93 and 101

2. Fll in the blanks.
   (a) 31 + ____ = 34   (d) 12 + ____ = 19   (g) 8 + ____ = 13
   (b) 5 + ____ = 9   (e) 59 + ____ = 62   (h) 55 + ____ = 63
   (c) 23 + ____ = 28   (f) 7 + ____ = 11   (i) 17 + ____ = 22

3. Find the difference by counting from the smaller number to "next TEN," and from "nect TEN" to larger number.
   (a) 8 and 15   (g) 28 and 87   (m) 17 and 54
   (b) 19 and 27   (h) 37 and 73   (n) 58 and 85
   (c) 16 and 23   (i) 19 and 66   (o) 59 and 83
   (d) 26 and 35   (j) 35 and 53   (p) 38 and 64
   (e) 28 and 44   (k) 66 and 81   (q) 56 and 65
   (f) 37 and 65   (l) 15 and 51   (r) 19 and 95

4. Have another person call out two numbers less than one hundred. Find the difference between them mentally. Repeat this exercise until you can do so effortlessly. *If possible, do this exercise on a turn about basis.*

5. Find the difference by using columns
   (a) 14 and 36   (g) 291 and 457   (m) 3,567,888,146 and 5,423,234,015
   (b) 24 and 65   (h) 485 and 549   (n) 6,012,345,678 and 9,876,543,210
   (c) 45 and 77   (i) 296 and 777   (o) 4,999,847,503 and 5,937,123,472
   (d) 21 and 33   (j) 267 and 444   (p) 3,567,888,146 and 5,423,234,015
   (e) 27 and 69   (k) 398 and 594   (q) 6,012,345,678 and 9,876,543,210
   (f) 33 and 75   (l) 89 and 777   (r) 4,999,847,503 and 5,937,123,472

Answer: **1.** (a) 5 (b) 5 (c) 6 (d) 7 (e) 6 (f) 8 (g) 5 (h) 9 (i) 8 **2.** (a) 3 (b) 4 (c) 5 (d) 7 (e) 3 (f) 4 (g) 5 (h) 8 (i) 5 **3.** (a) 7 (b) 8 (c) 7 (d) 9 (e) 16 (f) 28 (g) 59 (h) 36 (i) 47 (j) 18 (k) 15 (l) 36 (m) 37 (n) 27 (o) 24 (p) 26 (q) 9 (r) 76 **5.** (a) 22 (b) 41 (c) 32 (d) 12 (e) 42 (f) 42 (g) 166 (h) 64 (i) 481 (j) 177 (k) 196 (l) 688 (m) 1,855,345,869 (n) 3,864,197,532 (o) 937,275,969 (p) 1,855,345,869 (q) 3,864,197,532 (r) 937,275,969

Lesson A2.8   Word Problems

To solve a word problem one must translate it from English into a mathematical expression.

1. Some of the verbal expressions that may be translated as addition are as follows. *NOTE: The letter 'n' means a number.*

   The sum of a number and **20** .... $n + 20$
   10 plus a number .... $10 + n$
   The gain of **15** over a number .... $n + 15$
   A number increased by **9** .... $n + 9$
   A rise of **7** from a number .... $n + 7$
   a number expanded by **18** .... $n + 18$

30 more than a number .... n + 30
25 greater than a number .... n + 25
17 larger than a number .... n + 17
A number enlarged by **13** .... n + 13
A number augmented by **33** .... n + 33
A number grew by **51** .... n + 51

2. To solve a word problem one must be able to think logically with all the words used in describing that problem.

   (a) A certain auditorium has three sections. There are 1,032 seats in the center section and 584 seats in each of the side sections. How many people can be seated in the auditorium?

   SOLUTION:
   Number of seats in the center section   =   1,032
   Number of seats in the left section   =   584
   Number of seats in the right section   =   584
   Total number of seats   =   1032 + 584 + 584   =   2200
   Therefore, 2200 people can be seated in the auditorium.   Answer

   (b) In 1980, the highest populated states were: California (23,667,565), New York (17,558,072), and Texas (14,229,288). What was the combined population of these three states?
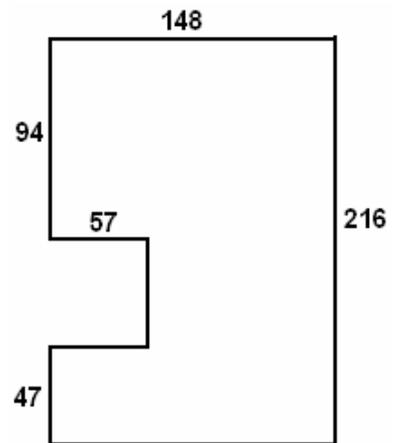
   SOLUTION:
   Combined population of these three states
   *   =   23,667,565 + 17,558,072 + 14,229,288*
   *   =   55,454,925   Answer*

   (c) John has $75, Jim has $18, and Marie has $12 more than John and Jim together. Find the total amount of money the three have together.

   SOLUTION:
   Amount of money John has   = $75
   Amount of money Jim has   = $18
   Amount of money Marie has
   *   = $12 + ($75 + $18)*
   *   = $105*
   The total amount of money the three have together
   *   = $75 + $18 + $105*
   *   = $198   Answer*

   (d) Find the distance around the building shown in the figure on the right (the dimensions are in feet).

   *SOLUTION:*
   The opposite sides of a rectangle are equal.
   Therefore, the distance around the building   =   148 + 216 + 148 + 216 + 57 + 57
   *   =   842 ft   Answer*
   Note that the numbers 94 and 47 are not used because they are part of the whole length (216 ft) of the building which is used twice, as is the width (148 ft).

☺ Exercise A2.8

1. Express the following mathematically.
   (a) **3** increased by the number.   (c) **8** larger than the number.
   (b) The number expanded by **17**.   (d) **5** more than the number.

2. Answer the following questions.
   (a) Joey has 6 pennies. He found 8 more. How many pennies does he have now?
   (b) Bill is 45 inches tall. Last summer he grew by 5 inches. How tall is he now?
   (c) There are 27 students in the class. 4 new students joined the class last week. How many students are in the class now?
   (d) John drove 90 miles one day and 135 miles the next day. What is the total distance that he traveled?
   (e) Billy weighs 86 lbs. Over Thanksgiving and Christmas holidays he gained another 15 lbs. How much does he weigh now.

3. The greatest known depth of our oceans is 36,198 feet, and the highest point on the earth is Mt. Everest, 29,028 feet. What is the vertical distance from the lowest point to the highest point on the earth?

4. The populations of some of the nations of the world in 1960 were: China, over 750,000,000; India, 439,235,082; Russia, 241,748,000; United States, 204,765,770. Find the combined population of China, India, Russia, and the United States.

5. Mary has $34, Jane has $15, and Helen has $27 more than Mary and Jane together. Find the total amount of money the three girls have together.

6. The figure shown on the right is called a triangle. The perimeter of a geometric figure is the sum of the lengths of all its sides. The word perimeter means "the measure around a figure." What is the perimeter of the triangle shown?

7. What is the perimeter of the figure shown on the right?

8. The greatest known depth of the Atlantic Ocean is 28,374 feet, and the highest point in Europe is 18,510 feet. What is the vertical distance between these two points?

9. John has $85; Bill has $92; and Lisa has $17 more than John and Bill together. Find the total amount of money the three of them have together.

10. The figure on the right is called a rectangle. What is the perimeter of the rectangle shown?
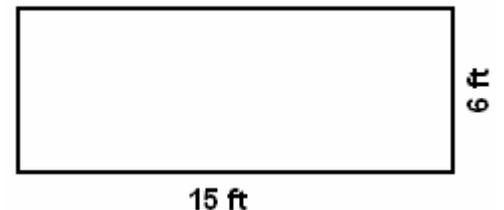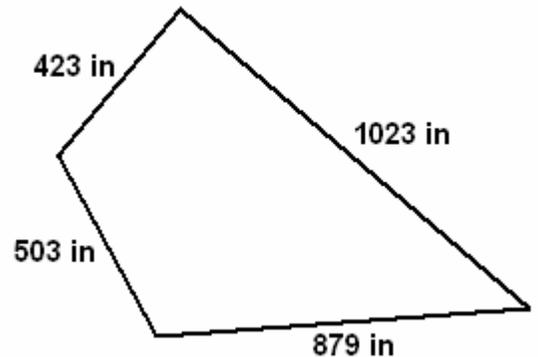
23 ft · 29 ft · 36 ft

423 in · 1023 in · 503 in · 879 in

6 ft · 15 ft

SUMMARY

ADDITION is counting together. In the initial stages one learns to add by counting more from a number using one's fingers. Then one learns the concept of "carry-overs" across the place values on abacus. With the knowledge of the numbering system one may then add single- and double-digit numbers rapidly in one's mind.

The following logic makes mental addition easy:

1. Adding two single digit numbers:

    (a) Start from the larger number. Count as many more as the smaller number.

    (b) If the sum goes beyond the next TEN, borrow from the second number to take the first number to TEN. Then add the remainder of the smaller number.

2. Adding a single-digit number to a double digit number:

    (c) Add the ONES to get the last digit of the sum. Then get the next larger number, which ends with that digit.

3. Adding two double- digit numbers:

    (d) Add to the first number, the TENS of the second number; and then ONES of the second number.

    (e) Get the sum of TENS. Then get the sum of ONES. Then add the two sums together.

4. Adding two numbers close to each other:

    (f) Double the smaller number; and then add the difference of the numbers

For addition of larger number one may use columns on paper. This still requires mental math to add each column. Calculators may be used as an aid, but mental awareness is still necessary to know if the calculated sum is correct. This awareness comes from practice with mental math.

Use of flash cards to memorize addition "facts" is not as effective as thinking systematically with numbers. It is, therefore, vital to learn the skills presented at this milestone.

DIAGNOSTIC TEST

1. Count the following together.
    (d) 7 horses and 5 horses   (d) 5 hours and 30 minutes
    (e) 7 horses and 5 camels   (e) 2 feet and 3 inches
    (f) 6 hours and 3 hours   (f) 2 inches and 3 inches

2. Solve the following problems.
    (d) For his birthday, Peter got 3 books from his uncle, and 3 books from his aunt. How many books did Peter receive on his birthday?
    (e) You caught 2 butterflies on Monday and 5 butterflies on Tuesday. How many butterflies did you catch in all?
    (f) Elli had 4 dolls. She got 5 more on her birthday. How many dolls does Elli have?

3. Add the following numbers by <u>counting more</u> on your fingers.
    (a) 9 + 3   (d) 5 + 23   (g) 8 + 37
    (b) 3 + 19   (e) 87 + 7   (h) 14 + 5
    (c) 9 + 20   (f) 9 + 99   (i) 6 + 86

4. Add the following on abacus.
    (a) 3 + 6   (d) 24 + 6   (g) 52 + 27
    (b) 9 + 3   (e) 44 + 3   (h) 79 + 21
    (c) 8 + 6   (f) 56 + 7   (i) 88 + 65

5. Add the following mentally.
    (d) 10 + 9   (d) 93 + 10   (g) 66 + 7   (j) 45 + 38
    (e) 20 + 5   (e) 64 + 9   (h) 48 + 5   (k) 79 + 14
    (f) 44 + 10   (f) 53 + 8   (i) 79 + 6   (l) 97 + 45

6. Count up to 20 by 2. (NOTE: This is counting 2, 4, 6… to 20.)

7. Count from 7 to 70 by 7.

8. Add the following mentally. Check your answer by adding in reverse order.
    (d) 2 + 3 + 8 + 7 + 6 + 4 + 8 + 9   = _____
    (e) 7 + 9 + 3 + 8 + 1 + 8 + 9 + 2   = _____
    (f) 3 + 6 + 6 + 4 + 1 + 2 + 7 + 8   = _____

9. Give the first ten counts by 12.

10. Give the first ten counts by 17.

11. Add the following by using columns on paper and pencil.
    (d) 463 + 359 + 706 = _____
    (e) 3,078 + 5,972 + 6,876 = _____
    (f) 23,087 + 15,223 + 309,854 = _____

GLOSSARY
[For additional words refer to the glossary at the end of Milestone # 1]

Addend   An **addend** is a number that is added to another number.

Addition   When we count two different quantities together we have **addition.** Only the quantities of the same unit may be added. To add, we start from the first number, and count as many more as the second number.

Associative   To **associate** is to accompany together. Therefore, **associative** describes the condition of accompanying or joining together.

We may associate numbers differently without changing the outcome in addition. This is called the <u>**Associative Property of Addition**</u>. Therefore, if **A, B** and **C** are any three numbers, then

$$A + (B + C) \quad = \quad (A + B) + C$$

Commutative   To **commute** is to change the order or arrangement of. Therefore, **commutative** describes interchange, or exchange.

We may add two numbers in any order. This is called the <u>**Commutative Property of Addition**</u>. Therefore, if **A** and **B** are any two numbers, then

$$A + B \quad = \quad B + A$$

Equal Sign   An **equal sign (=),** expresses the equality between two expressions.

Plus Sign   A **plus (+),** when placed between two numbers, expresses that those two numbers are to be added.

Property   A **property** is a basic or essential attribute, such as, the physical properties of matter.

Storage count   The "storage count," is that count, which when combined with a single-digit number, takes it to 10. For example, the 'storage count' for 6 is 4, because 4 combined with 6 takes it to 10. "Storage count" is represented by the number of beads at the storage end of the abacus.

Sum   A **sum** is the result of addition.

Turn about   To switch around the roles of coach and student while doing an exercise.

Word Problem   A **word problem** is statement of a problem in English that may be solved numerically by translating it first into the language of mathematics.