

**MATH MILESTONE # 1****NUMBERS & PLACE VALUES**

The word, **milestone**, means “a point at which a significant (important, of consequence) change occurs.” A Math Milestone refers to a significant point in the understanding of mathematics.

**To reach this milestone one should be able to read and write numbers into the billions and more.**

The word, **datum**, means “a single piece of information.” Each datum given here is accompanied by a diagnostics, which helps locate any blanks in understanding.

<b><u>Datum</u></b>	<b><u>Page</u></b>
1.1 Mathematics starts with counting.....	3
1.2 Mathematics is a learning tool, which teaches us how to think systematically.....	5
1.3 A UNIT is what we count one at a time. A NUMBER is how many we have counted .....	6
1.4 Abacus provides a systematic way of counting. Zero is an “absence of count” .....	7
1.5 On abacus, counts of ten beads on a wire are regrouped as a count of one bead on the next wire .....	9
1.6 Different DIGITS are a short hand for different number of beads on a wire .....	11
1.7 Numbers beyond nine have a digit for TENS in addition to a digit for ONES .....	12
1.8 Numbers beyond ninety-nine have an additional digit for HUNDREDS .....	15
1.9 Digits are like “letters.” Numbers are like “words.” The place values of ONE, TEN and HUNDRED form a group.....	17
1.10 The Group of THOUSANDS consists of ONE thousand, TEN thousands, and HUNDRED thousands .....	19
1.11 The Group of MILLIONS consists of ONE million, TEN millions, and HUNDRED millions .....	21
1.12 The Group of BILLIONS consists of ONE billion, TEN billions, and HUNDRED billions .....	23
Diagnostic Test .....	25
Summary .....	26
Glossary .....	27

Get a simple ABACUS, as described in Lesson 1.4. Alternately, you may access the **VIRTUAL ABACUS** at <http://www.mathfundamentals.org/abacus.htm> for the purpose of this milestone.

Please consult the **Glossary** supplied with this Milestone for mathematical terms. Consult a regular dictionary at [www.dictionary.com](http://www.dictionary.com) for general English words that one does not understand fully.

**Do the Diagnostics in the sequence given. Then study the lessons, as necessary.**

Researched and written by Vinay Agarwala  
Edited by Ivan Dorskocil

## MATH MILESTONE # 1

### NUMBERS & PLACE VALUES

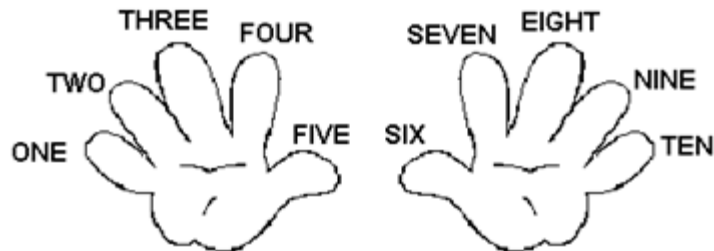
☺ **Datum 1.1:** *Mathematics starts with counting.*

#### ***Diagnostics***

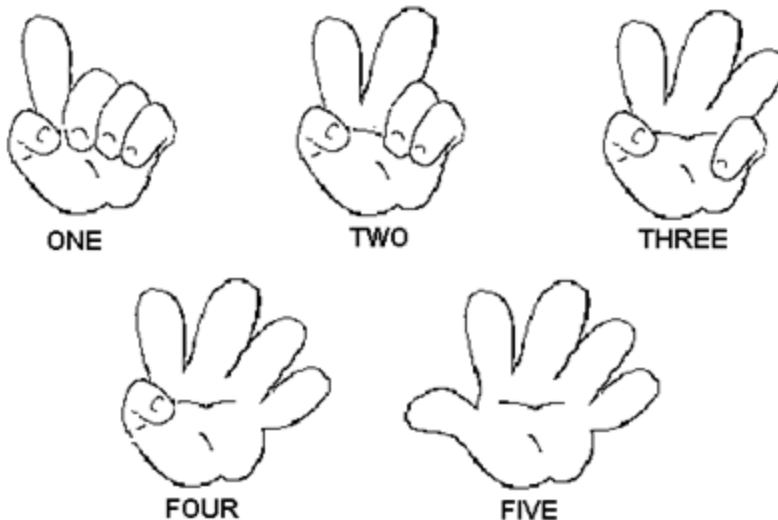
1. Count the fingers on your two hands. How many fingers are there?
2. How will you show the number SEVEN using your fingers?

Answer: 1. Normally there are ten fingers. 2. See Lesson 1.1

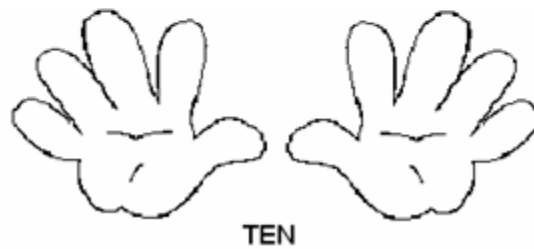
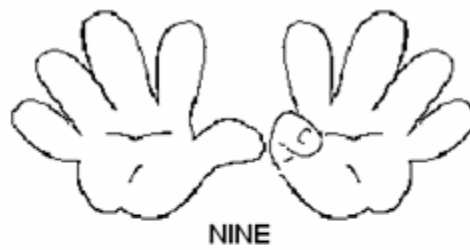
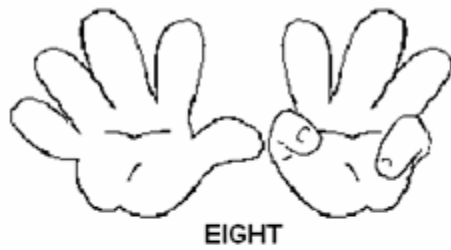
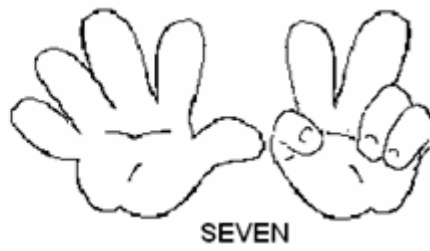
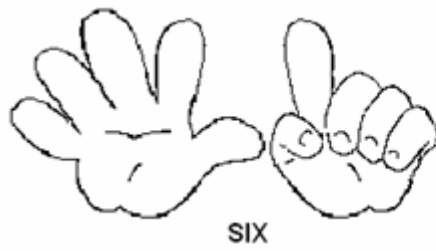
**Lesson 1.1:** Study the following as determined from Diagnostics.



1. In counting, we call out the first item as ONE, the next item as TWO, the next item as THREE, and so on. **NOTE: The counting numbers are also referred to as Natural Numbers.**
2. The following finger configurations have been used to represent the counts from ONE to FIVE using one hand.



3. The following finger configurations have been used to represent the counts from SIX to TEN using both hands.



☺ **Datum 1.2:** *Mathematics is a learning tool, which teaches us how to think systematically.*

### ***Diagnostics***

1. **How does counting show us what Mathematics is?**
2. **What can we learn from Mathematics?**

Answer: 1. The word MATHEMATICS means, "learning." Counting helps us learn how many things there are. 2. We can learn how to think systematically from Mathematics.

**Lesson 1.2:** Study the following as determined from Diagnostics.

1. When we count, we learn how many things are there. Thus, counting may be looked upon as a tool for learning. The word MATHEMATICS comes from a Greek word, *mathema*, which means, "learning." **Mathematics, essentially, is a tool for learning.**
2. One counts in the sequence: ONE, TWO, THREE, FOUR, FIVE, etc. always. When one knows the system of counting one can easily tell the next number. This is systematic learning, which reduces the need to memorize. **Mathematics teaches us to think systematically.**

☺ **Datum 1.3:** *A UNIT is what we count one at a time. A NUMBER is how many we have counted.*

### ***Diagnostics***

1. What is the unit when you are counting pennies?
2. What is the unit when you are counting hundred-dollar bills?
3. Identify which of the following are numbers and which are units.  
(a) Dollar (b) Three (c) Cup (d) Group (e) Ten (f) Cat (g) Seven

ANSWER: 1. A penny 2. A one hundred-dollar bill 3. (a) Unit (b) Number (c) Unit (d) Unit (e) Number (f) Unit (g) Number

**Lesson 1.3:** Study the following as determined from Diagnostics.

1. A UNIT is “what we count one at a time.”
  - (a) When we count fingers one at a time, each finger is a unit.
  - (b) When we count chairs one at a time, each chair is a unit.
  - (c) When we count ten-dollar bills one at a time, each ten-dollar bill is a unit.
2. A NUMBER is “how many we have counted.”
  - (a) When we say, “eight fingers,” the number is EIGHT and the unit is A FINGER.
  - (b) When we say, “six chairs,” the number is SIX and the unit is A CHAIR.
  - (c) When we say, “three ten-dollar bills,” the number is THREE and the unit is a ten-dollar bill.

☺ **Datum 1.4:** *Abacus provides a systematic way of counting. Zero is an “absence of count.”*

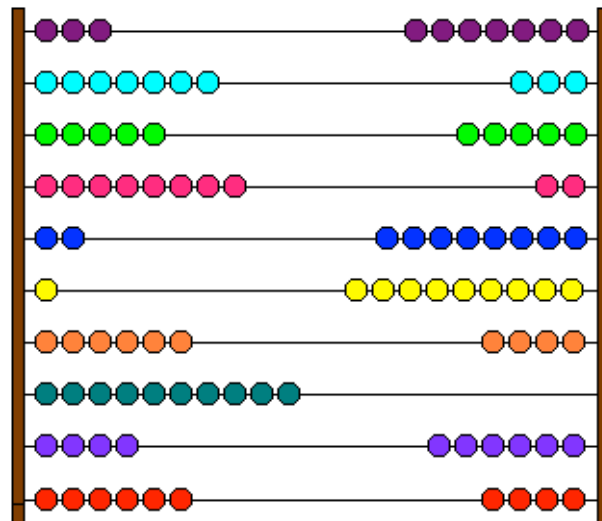
**Diagnostics**

1. On an Abacus, point to the beads counted; point to the beads in storage.
2. Show the following counts on abacus.  
(a) Seven (b) four (c) Nine
3. What is zero? Show zero on abacus.

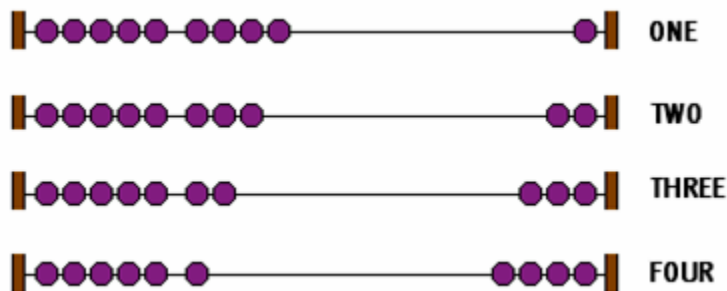
Answer: 3. ZERO is “absence of count.” For ZERO, all the beads are pushed to the left, i.e. placed in storage on abacus.

**Lesson 1.4:** Study the following as determined from Diagnostics.

1. An ABACUS is a counting board with ten wires and ten beads on each wire. The word ABACUS comes from a word meaning, “a board sprinkled with dust for writing.”



2. We count by moving beads to the right one at a time as follows.







☺ **Datum 1.5:** *On abacus, counts of ten beads on a wire are regrouped as a count of one bead on the next wire.*

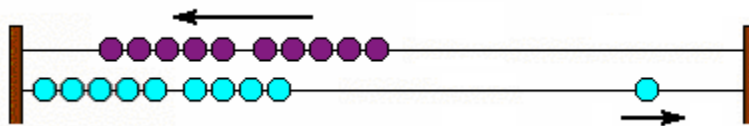
**Diagnostics**

1. Show ten on abacus by counting on first wire.
2. Show ten on abacus after regrouping to next wire.
3. Count on abacus from eight to twelve demonstrating the rule of regrouping.

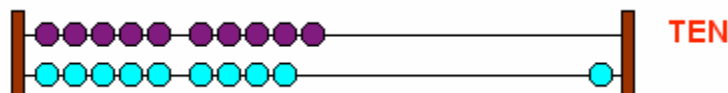
ANSWER: See Lesson 1.5

**Lesson 1.5:** Study the following as determined from Diagnostics.

1. "Counts of ten beads on a wire are regrouped as count of one bead on the next wire."



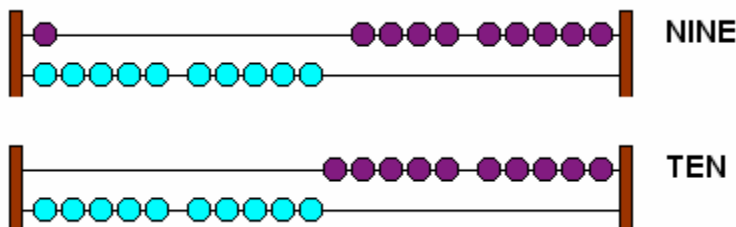
Therefore, we show TEN on abacus not with ten beads on the first wire, but with one bead on the second wire.

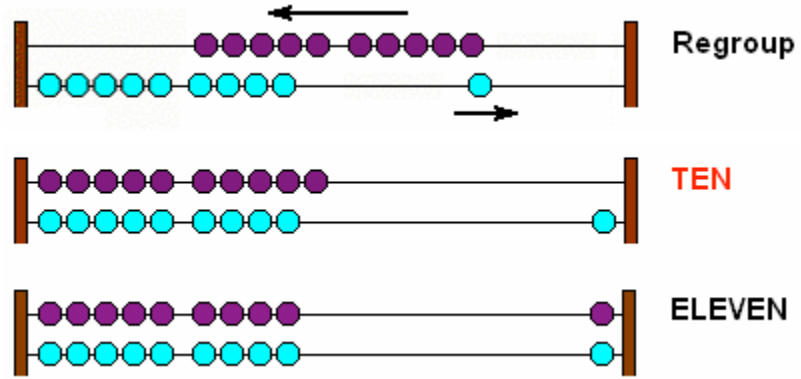


This is like regrouping 10 pennies into a dime.



2. Therefore, we count from nine to eleven as follows.





☺ **Datum 1.6:** *Different DIGITS are short hand for different number of beads on a wire.*

### ***Diagnostics***

1. What is the idea behind DIGITS?
2. How many different digits are there?
3. How many digits does it take to write the number ten?

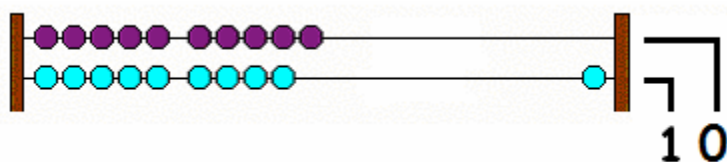
Answer: 1. The digits correspond to the number of beads on a wire. 2. There are ten different digits — 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. 3. Two digits — "1" and "0."

**Lesson 1.6:** Study the following as determined from Diagnostics.

1. Digits provide shorthand on paper for how many beads are counted on a wire of abacus. The digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 as follows.

0	ZERO	_____
1	ONE	_____●
2	TWO	_____●●
3	THREE	_____●●●
4	FOUR	_____●●●●
5	FIVE	_____●●●●●
6	SIX	_____●●●●●●
7	SEVEN	_____●●●●●●●
8	EIGHT	_____●●●●●●●●
9	NINE	_____●●●●●●●●●

2. There is no new digit needed for TEN because it can be written with existing digits after the Rule of Regrouping is applied. TEN is written with two digits as "10."



☺ **Datum 1.7: Numbers beyond nine have a digit for TENS in addition to a digit for ONES.**

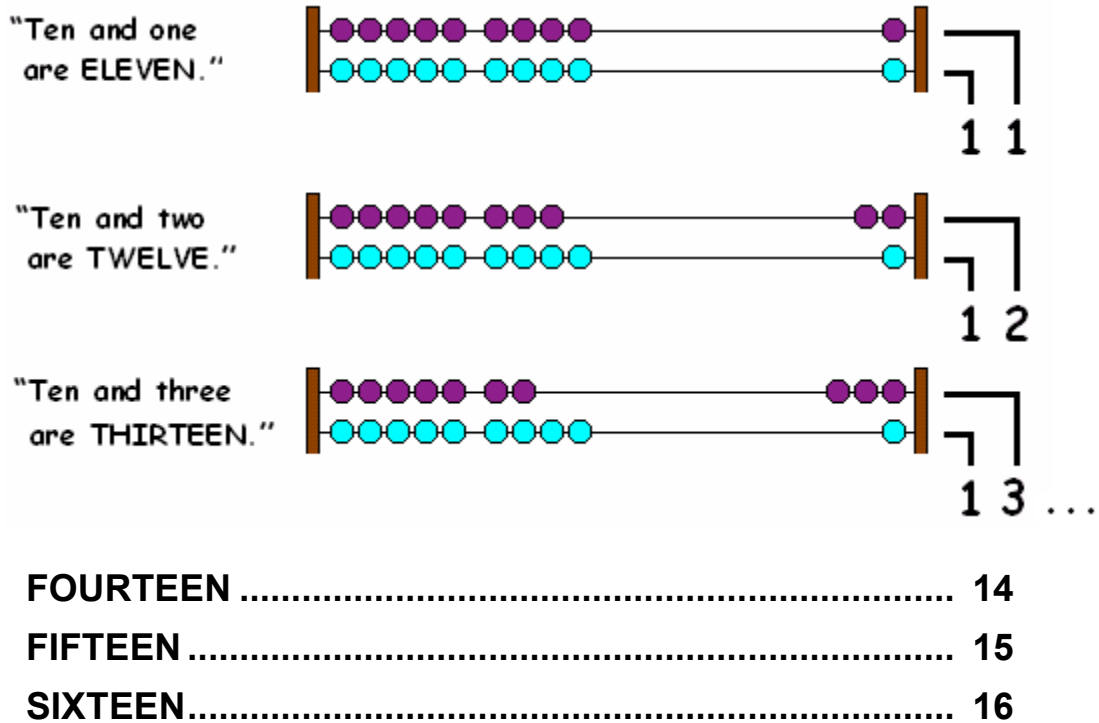
**Diagnostics**

1. The number “sixteen” is made up of \_\_\_ TEN and \_\_\_ ONES.
2. The number “seventy-five” is made up of \_\_\_ TENS and \_\_\_ ONES.
3. Show the following counts on abacus. Write them down using digits.  
 (a) Thirteen (b) Seventeen (c) Twenty (d) Fifty (e) Thirty-seven  
 (f) Eighty (g) Seventy-three (h) Ninety-seven
4. Read the following numbers:  
 (a) 12 (b) 15 (c) 19 (d) 55 (e) 83 (f) 70 (g) 49 (h) 94

Answer: 1. 1 TEN and 6 ONES. 2. 7 TENS and 5 ONES. 3. (a) 13 (b) 17 (c) 20 (d) 50 (e) 37 (f) 80 (g) 73 (h) 97. 4. (a) Twelve (b) Fifteen (c) Nineteen (d) Fifty-five (e) Eighty-three (f) Seventy (g) Forty-nine (h) Ninety-four

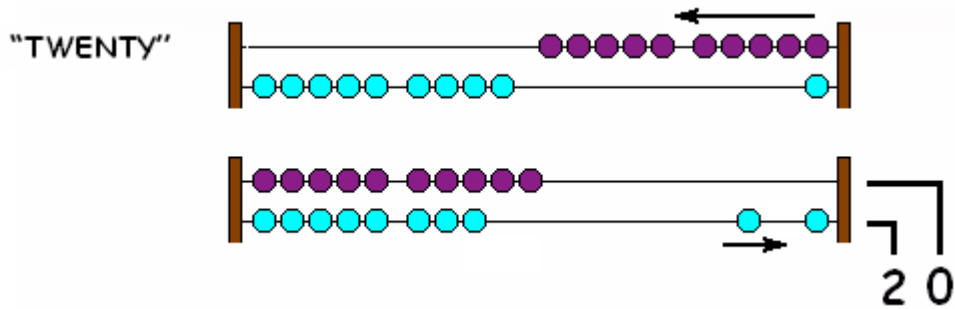
**Lesson 1.7:** Study the following as determined from Diagnostics.

1. The next count after ten is “ten and one,” or ELEVEN. This will be one bead on the second wire (TENS), and one bead on the first wire (ONES). For subsequent counts add beads on the first wire. The counts after ten are:

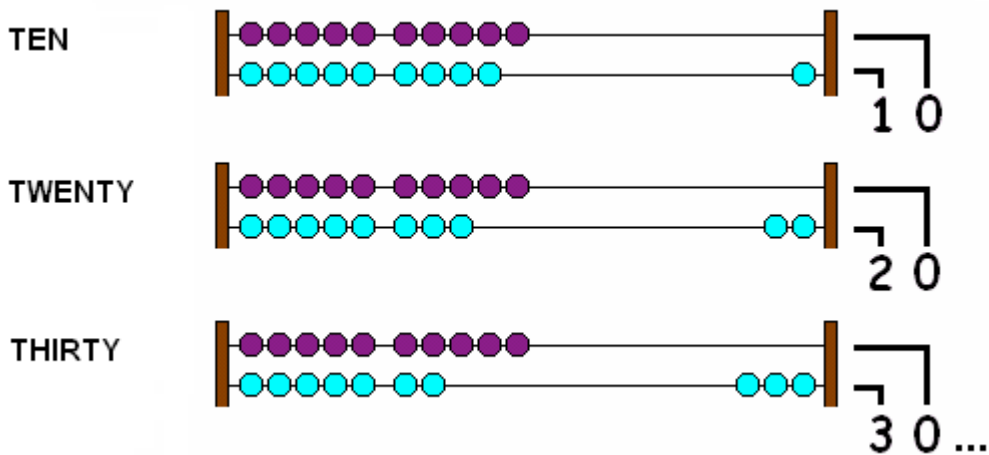


**SEVENTEEN** ..... 17  
**EIGHTEEN** ..... 18  
**NINETEEN** ..... 19

2. At count TWENTY; once again we regroup the “ten ONES” on the first wire, as “one TEN” on the second wire. TWENTY is, therefore, written as “20.”



3. Each bead on the second wire is a TEN. We count on the second wire by TENS as, **Ten, twenty, thirty, forty fifty, sixty, seventy, eighty, and ninety**

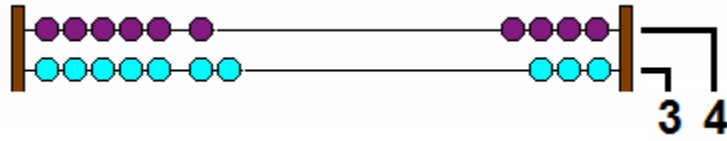


**FORTY** ..... 40  
**FIFTY** ..... 50  
**SIXTY** ..... 60  
**SEVENTY** ..... 70  
**EIGHTY** ..... 80  
**NINETY** ..... 90

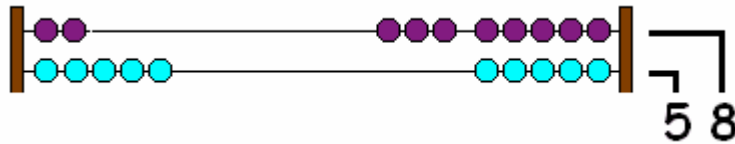
4. Please note that each bead on the first wire is a ONE; and we count the ONES as, **One, two, three, four, five, six, seven, eight, and nine**

5. Thus, we may show numbers as a combination of TENS and ONES.

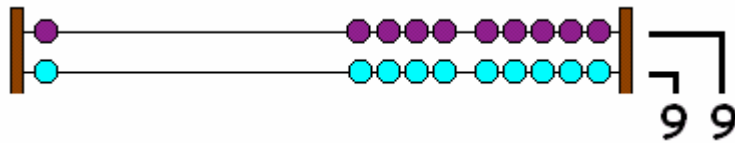
(a) The number THIRTY-FOUR is made up of 3 TENS and 4 ONES.



(b) The number FIFTY-EIGHT is made up of 5 TENS and 8 ONES.



(c) The number NINETY-NINE is made up of 9 TENS and 9 ONES.



☺ **Datum 1.8: Numbers beyond ninety-nine have an additional digit for HUNDREDS.**

**Diagnostics**

- “Three hundred seven” is made up of \_\_\_ HUNDREDS, \_\_\_ TENS and \_\_\_ ONES.
- Show the following counts on abacus, and then write them down using digits.
 

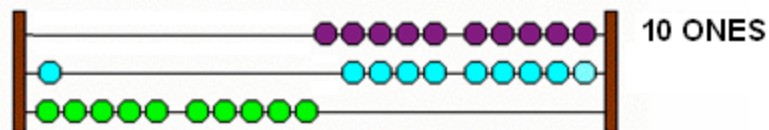
(a) Three hundred twelve	(d) Five hundred eighty
(b) Three hundred twenty-one	(e) Five hundred eight
(c) Three hundred five	(f) Nine hundred
- Read the following numbers:
 

(a) 111	(b) 277	(c) 658	(d) 704	(e) 410
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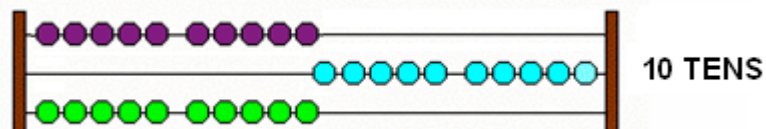
Answer: 1. 3 HUNDREDS, 0 TENS and 7 ONES. 2. (a) 312 (b) 321 (c) 305 (d) 580 (e) 508 (f) 900 3. (a) One hundred eleven (b) Two hundred seventy-seven (c) Six hundred fifty-eight (d) Seven hundred four (e) Four hundred ten

**Lesson 1.8:** Study the following as determined from Diagnostics.

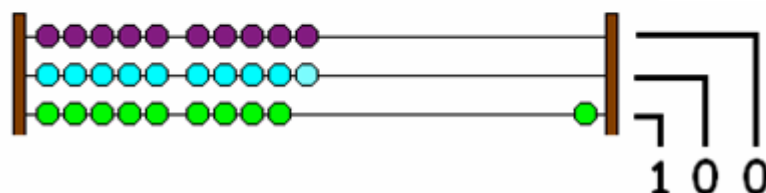
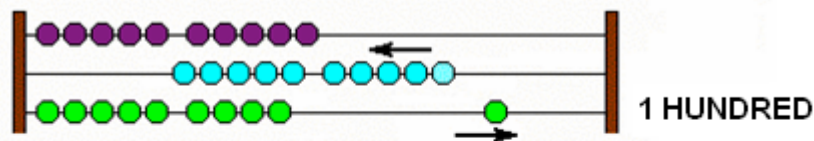
- One more count from NINETY-NINE gives us all the ten beads to the right on the first wire.



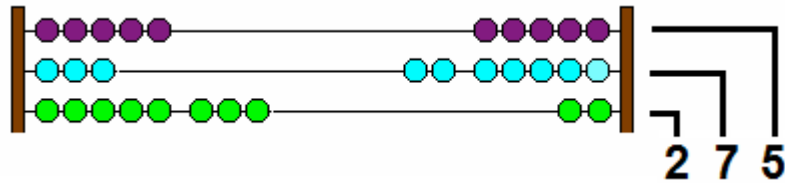
- We apply the Rule of Regrouping and regroup them as one bead on the next wire. This gives us all the ten beads to the right on the second wire.



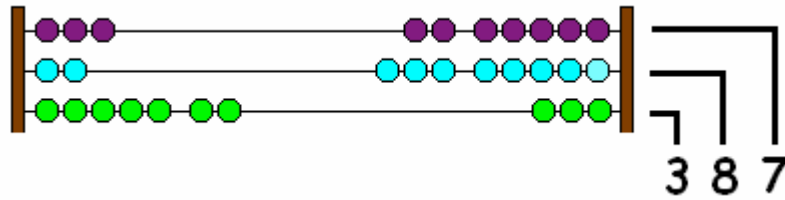
- We apply the Rule of regrouping again and regroup them as one bead on the next wire. This gives us one bead to the right on the third wire. This we call “one hundred.”



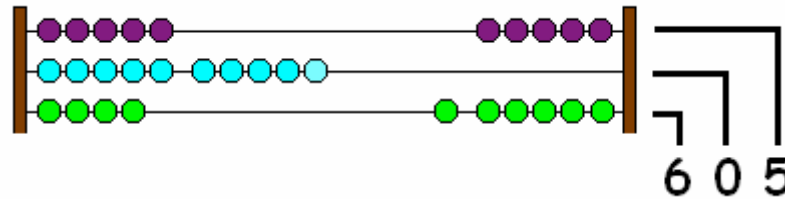
4. Each bead on the third wire is a HUNDRED. We count on the third wire by HUNDREDS as,  
***One hundred, two hundreds, three hundreds, four hundreds, five hundreds, and so on...***
5. Thus, we may show numbers as a combination of HUNDREDS, TENS and ONES.
  - (a) The number “Two Hundred Seventy-Five” is made up of 2 HUNDREDS, 7 TENS and 5 ONES.



- (b) The number “Three Hundred Eighty-Seven” is made up of 3 HUNDREDS, 8 TENS and 7 ONES.



- (c) The number “Six Hundred Five” is made up of 6 HUNDREDS, 0 TENS and 5 ONES.





☺ **Datum 1.9:** *Digits are like “letters.” Numbers are like “words.” The place values of ONE, TEN and HUNDRED form a group.*

**Diagnostics**

1. What is the difference between a digit and a number?
2. How many different digits are there in our numbering system?
3. Is 7 a digit or a number?
4. How many single-digit numbers are there?
5. How many double-digit numbers are there?
6. What are the smallest and largest three-digit numbers?

Answer: 1. A digit is like a letter, whereas, a number is like a word. 2. There are ten different digits---0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. 3. 7 is a single-digit number. 4. Ten if you include 0. 5. Ninety. 6. 100, and 999

**Lesson 1.9:** Study the following as determined from Diagnostics.

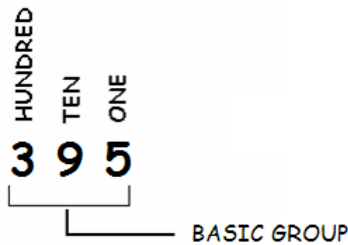
1. Digits are used to write numbers, just like letters are used to write words. The number “three hundred ninety-five” is written with three digits: 3, 9, and 5.



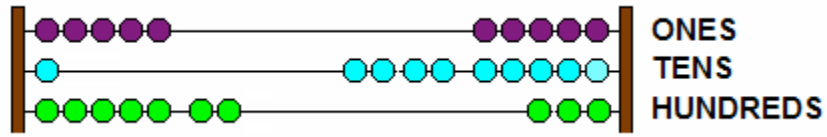
**5** is a number written with one digit, just like **I** is a word written with one letter.  
**35** is a number written with two digits, just like **ME** is a word written with two letters.  
**164** is a number written with three digits, just like **YOU** is a word written with three letters.

From **0 to 9** we have single-digit numbers. There are nine single-digit numbers and zero.  
 From **10 to 99** we have double-digit numbers. There are ninety double-digit numbers.  
 From **100 to 999** we have 3-digit numbers. There are nine hundred 3-digit numbers.

2. The digits in a number has have *place values* of ONE, TEN and HUNDRED from right to left. The number “three hundred ninety-five” is made up 5 ONES, 9 TENS, and 3 HUNDREDS. These place values form a basic group of three.



3. The place values of ONE, TEN, and HUNDRED correspond to the first, second, and third wires of the abacus.



Note that the same three colors may be used consistently for ONES, TENS, and HUNDREDS, in this and subsequent groups.

4. From one place value to the next, the value increases by a factor of TEN.

☺ **Datum 1.10:** *The Group of Thousands consists of ONE thousand, TEN thousands, and HUNDRED thousands. We will call Group of Thousands just THOUSANDS.*

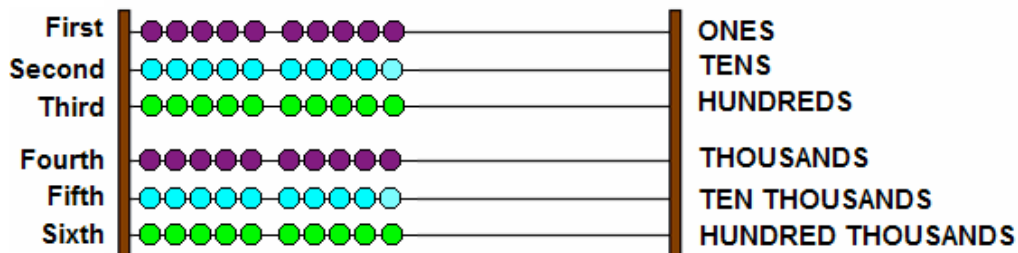
**Diagnostics**

1. Place a comma to separate thousands from the basic group.  
 (a) 3829    (b) 56942    (c) 419736    (d) 100001    (e) 350093
2. Read the following numbers.  
 (a) 1,111                      (c) 532,658                      (e) 300,005  
 (b) 23,277                      (d) 500,074                      (f) 101,010
3. Write the following numbers using digits.  
 (a) Six thousand, three hundred sixty-five  
 (b) Ninety Eight thousand, eight hundred one  
 (c) Two hundred sixty thousand, four hundred twenty seven  
 (d) Nine hundred thousand, ninety-nine  
 (e) Three hundred twenty-nine thousand, five hundred forty-two  
 (f) Seventy-seven thousand, six hundred  
 (g) Four hundred thousand, five  
 (h) Two hundred thirteen thousand, eighty-six  
 (i) Six hundred six thousand, sixty-six

Answer: 1. (a) 3,829 (b) 56,942 (c) 419,736 (d) 100,001 (e) 350,093 2. (a) One thousand, one hundred eleven (b) Twenty-three thousand, two hundred seventy-seven (c) Five hundred thirty-two thousand, six hundred fifty-eight (d) Five hundred thousand, seventy-four (e) Three hundred thousand, five (f) One hundred one thousand, ten 3. (a) 6,365 (b) 98,801 (c) 260,427 (d) 900,099 (e) 329,542 (f) 77,600 (g) 400,005 (h) 213,086 (i) 606,066

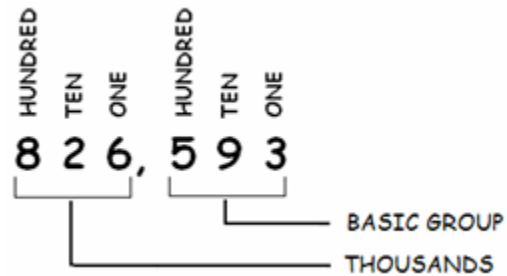
**Lesson 1.10:** Study the following as determined from Diagnostics.

1. The Rule of Regrouping applies to digits beyond the THOUSANDS as follows.  
 10 HUNDREDS may be exchanged for 1 THOUSAND on fourth wire.  
 10 THOUSANDS may be exchanged for 1 TEN THOUSAND on fifth wire.  
 10 TEN THOUSANDS may be exchanged for 1 HUNDRED THOUSAND on sixth wire.



2. Thus, the THOUSANDS is a group of ONE, TEN, and HUNDRED. A comma is used to separate the thousands group from the basic group. For example, the following number is made up of

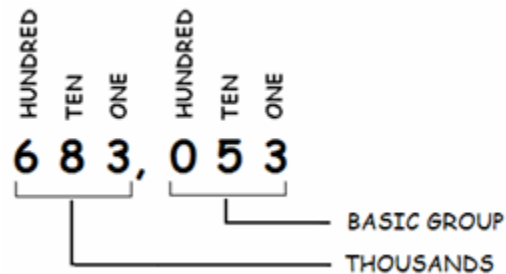
8 hundred thousands  
 2 ten thousands  
 6 one thousands  
 5 hundreds  
 9 tens  
 3 ones



We read this number as: Eight hundred twenty-six thousand, five hundred ninety-three.

3. **When no count exists for a place value, a zero is placed there.** For example, in the following number the place value of HUNDRED is missing. This number is written as shown below.

6 hundred thousands  
 8 ten thousands  
 3 one thousands  
 0 hundreds  
 5 tens  
 3 ones



We read this number as: Six hundred eighty-three thousand, fifty-three,

☺ **Datum 1.11:** *The Group of Millions consists of ONE million, TEN millions, and HUNDRED millions. We will call Group of Millions just MILLIONS.*

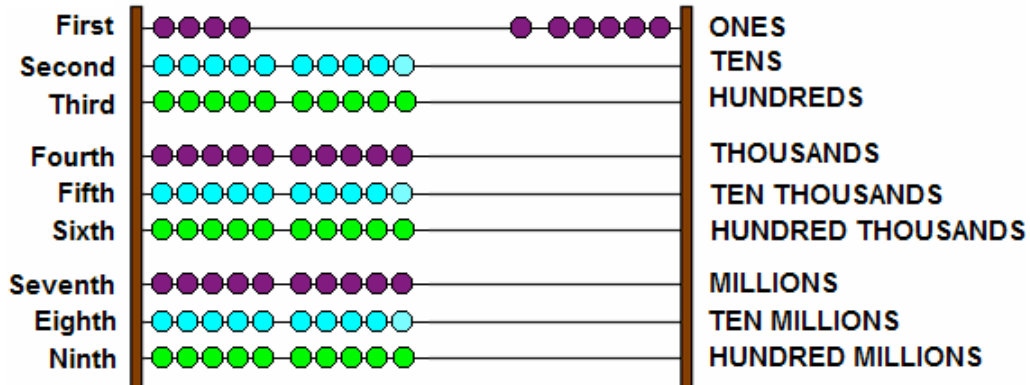
### Diagnosics

- Place commas at the correct place in the following numbers  
 (a) 8268268      (c) 826826826      (e) 305009023  
 (b) 82682682      (d) 100000000
- Read the following numbers.  
 (a) 5,762,869      (c) 765,532,658      (e) 9,009,009  
 (b) 27,045,008      (d) 300,006,074      (f) 590,008,060
- Write the following numbers.  
 (a) Two million, three hundred four thousand, five hundred sixteen  
 (b) Forty-five million, four hundred sixty-four thousand, eight hundred one  
 (c) Two hundred sixty million, thirty-six thousand, four hundred twenty seven  
 (d) Eight million, seven thousand, ninety-nine  
 (e) Six hundred forty-three million, eighty-six  
 (f) Sixty-four million, two hundred six thousand  
 (g) One hundred eleven million, two hundred fifty- four thousand, five  
 (h) Nineteen million, nine hundred thousand, nineteen  
 (i) One hundred sixty million, six

Answer: 1. (a) 8,268,268 (b) 82,682,682 (c) 826,826,826 (d) 100,000,000 (e) 305,009,023  
 2. (a) Five million, seven hundred sixty-two thousand, eight hundred sixty-nine  
 (b) Twenty-seven million, forty-five thousand, eight (c) Seven hundred sixty-five  
 million, five hundred thirty-two thousand, six hundred fifty-eight (d) Three hundred  
 million, six thousand, seventy-four (e) Nine million, nine thousand, nine (f) Five  
 hundred ninety million, eight thousand, six hundred thirty 3. (a) 2,304,516 (b) 45,464,801  
 (c) 260,036,427 (d) 8,007,099 (e) 643,000,086 (f) 64,206,000 (g) 111,254,005  
 (h) 19,900,019 (i) 160,000,006

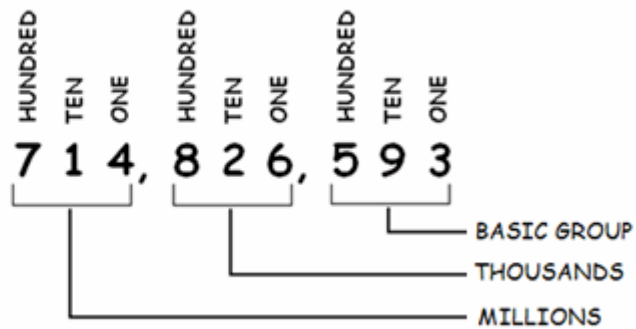
**Lesson 1.11:** Study the following as determined from Diagnostics.

- The Rule of Regrouping applies to digits beyond the THOUSANDS as follows.  
 10 HUNDRED THOUSAND may be exchanged for 1 MILLION on seventh wire.  
 10 MILLIONS may be exchanged for 1 TEN MILLION on eighth wire.  
 10 TEN MILLIONS may be exchanged for 1 HUNDRED MILLION on ninth wire.



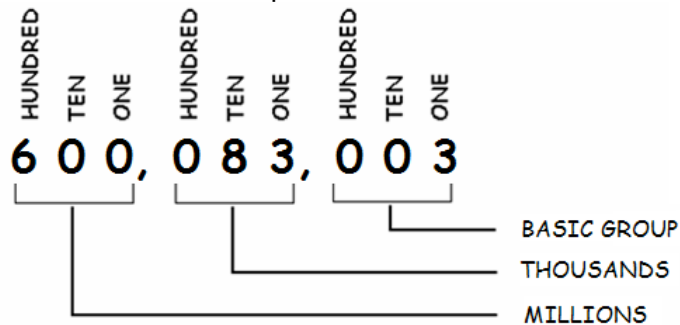
2. Thus, the MILLIONS is a group of ONE, TEN, and HUNDRED. A comma is used to separate the millions group from the thousands group. For example, the following number is made up of

- 7 hundred millions
- 1 ten millions
- 4 one millions
- 8 hundred thousands
- 2 ten thousands
- 6 one thousands
- 5 hundreds
- 9 tens
- 3 ones



We read this number as: Seven hundred fourteen million, eight hundred twenty-six thousand, five hundred ninety-three.

3. The following number is made up of 600 MILLIONS, 83 THOUSANDS, and 3. When no count exists for a place value, a zero is placed there.



We read this number as: six hundred million, eighty-three thousand, three.

- ☺ **Datum 1.12:** *The Group of Billions consists of ONE billion, TEN billions, and HUNDRED billions. We will call Group of Billions just BILLIONS.*

### Diagnosics

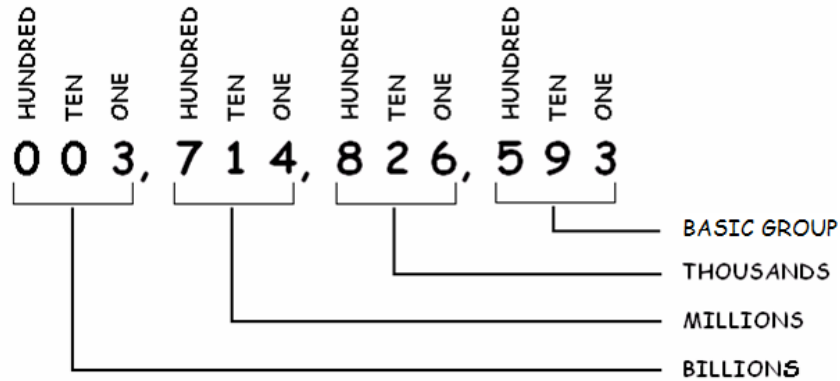
- Place commas at the correct place in the following numbers  
 (a) 8268268031      (c) 826682360826      (e) 302500943023  
 (b) 82682682562      (d) 100000000000
- Read the following numbers.  
 (a) 1,002,002,009      (c) 249,765,532,658      (e) 9,009,009,009  
 (b) 38,027,045,008      (d) 302,241,006,074      (f) 300,590,008,060
- Write the following numbers.  
 (a) 6 billion, 425 million, 606 thousand, three hundred four  
 (b) 25 billion, 43 million, 60 thousand, fifty  
 (c) 793 billion, 446 million, 237 thousand, five hundred sixty-five  
 (d) 100 billion, 3 million, 2 thousand, one  
 (e) 27 billion, 67 million, 35 thousand, eighty-seven  
 (f) One billion, five million, six  
 (g) Ten billion, one hundred million, fifty-five thousand, twelve  
 (h) Six hundred six billion, three hundred forty-one  
 (i) Three hundred billion, forty million, thirty-four

Answer: 1. (a) 8,268,268,031 (b) 82,682,682,562 (c) 826,682,360,826 (d) 100,000,000,000  
 (e) 302,500,943,023 2. (a) 1 billion, 2 million, 2 thousand, 9 (b) 38 billion, 27 million,  
 45 thousand, 8 (c) 249 billion, 765 million, 532 thousand, 658 (d) 302 billion, 241  
 million, 6 thousand, 74 (e) 9 billion, 9 million, 9 thousand, 9 (f) 300 billion, 590 million,  
 8 thousand, 60 3. (a) 6,425,606,304 (b) 25,043,060,050 (c) 793,446,237,565  
 (d) 100,003,002,001 (e) 27,067,035,087 (f) 1,005,000,006 (g) 10,100,055,012  
 (h) 606,000,000,341 (i) 300,040,000,034

**Lesson 1.12:** Study the following as determined from Diagnostics.

- The Rule of Regrouping applies to digits beyond the MILLIONS as follows.  
 10 HUNDRED MILLIONS may be exchanged for 1 BILLION on the tenth wire.  
 10 BILLIONS may be exchanged for 1 TEN BILLION.  
 10 TEN BILLIONS may be exchanged for 1 HUNDRED BILLION.

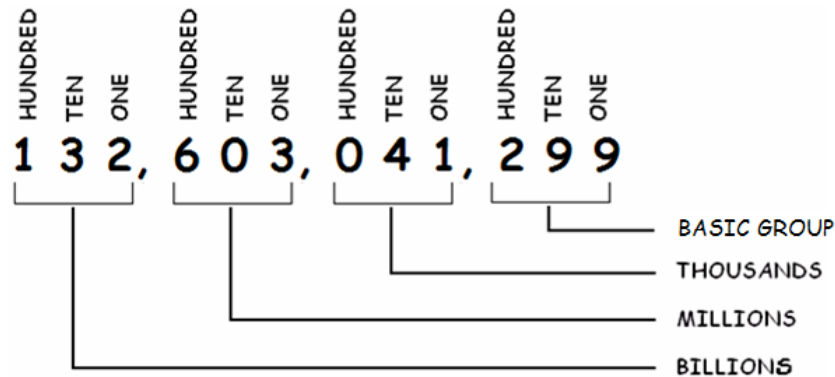
Note that, on abacus, three colors may be used consistently for ONES, TENS, and HUNDREDS, in all groups.



2. This number above is made up of 3 BILLIONS, 714 MILLIONS, 826 THOUSANDS, and 593. It is read as three billion, seven hundred fourteen million, eight hundred twenty-six thousand, five hundred ninety-three.

Thus, the BILLIONS is a group of ONE, TEN, and HUNDRED. A comma is used to separate the billions group from the millions group. For example, the following number

3. The number below is made up of 132 BILLIONS, 603 MILLIONS, 41 THOUSANDS, and 299. It is read as one hundred thirty-two billion, six hundred three million, forty-one thousand, two hundred ninety-nine.



Note that no place value is skipped. Since there are no counts for ten million, and hundred thousand, zeros are placed there as a place-holder.

4. Beyond BILLIONS we have groups of

TRILLIONS  
 QUADRILLION  
 QUINTILLION  
 SEXTILLION  
 SEPTILLION  
 OCTILLION  
 NONILLION  
 DECILLION, etc.



## DIAGNOSTIC TEST

**Before you proceed to Milestone #2, go through the following diagnostic test to find out if you understand the material of Milestone #1 fully.**

1. Read the following numbers. (Lessons 1.9 to 1.12)
 

(a) 25,807	(d) 3,000,009,133
(b) 357,000	(e) 325,601,213,000,102
(c) 3,007,002	
2. Write the following numbers. (Lessons 1.9 to 1.12)
  - (a) Seventy-seven thousand, three hundred nine
  - (b) Forty-two thousand, seven
  - (c) Six million, sixty-six thousand, sixty
  - (d) Four hundred forty-four thousand, four
  - (e) One hundred eight billion, four hundred fifty-six million, eighty-seven
3. Which of the following are units, and which are numbers? (Lesson 1.3)
 

(a) Dollar (b) Ten (c) Ten-dollar bill (d) Group (e) Six (f) Cat (g) One
4. Which of the following are natural numbers? (Lesson 1.1)
 

(a) 5 (b)  $\frac{1}{2}$  (c) 17 (d) -6 (e) 0 (f) 8 (g)  $\frac{1}{4}$
5. Is 10 a digit or a number? (Lesson 1.9)
6. Is 5 a digit or a number? (Lesson 1.9)
7. What are the place values of the underlined digits in the following numbers. (Lessons 1.9 to 1.12)
 

(a) 10 (b) 145 (c) 3,257 (d) 5,000,400 (e) 31,952,833

ANSWER: 1. (a) Twenty-five thousand, eight hundred seven (b) Three hundred fifty-seven thousand (c) Three million, seven thousand, two (d) Three billion, nine thousand, one hundred thirty-three (e) Three hundred twenty-five trillion, six hundred one billion, two hundred thirteen million, one hundred two 2. (a) 77,309 (b) 42,007 (c) 6,066,060 (d) 444,004 (e) 108,456,000,087 3. (a) Unit (b) Number (c) Unit (d) Unit (e) Number (f) Unit (g) Number 4. (a) Yes (b) No (c) Yes (d) No (e) No (f) Yes (g) No 5. It is a number written with two digits. 6. It is a number written with one digit. 7. (a) Ten (b) One Hundred (c) One (d) One Hundred (e) One Million

**If you failed to answer any question correctly, go back and restudy the corresponding lesson(s) marked at the question.**

## SUMMARY

The purpose of **Mathematics** is help one learn to think and reason in a systematic manner. This starts with learning to think systematically with numbers. The first part of Mathematics is called **Arithmetic**. The word ARITHMETIC (*arithmos* number + *techne* skill) means, “Skill with numbers.”

Arithmetic helps us determine “how many” or “how much” of something. Therefore, it introduces the ideas of unit, number and place values. The first action of Arithmetic is **counting**. Arithmetic builds upon the concept of place values to develop a number logic that helps solve problems mentally. The “logic” of place values is expressed in the **Rule of Regrouping** as follows.

WHENEVER ALL THE BEADS ARE TO THE RIGHT ON A WIRE, THEY ARE RETURNED TO THE LEFT AND REPLACED BY ONE BEAD TO THE RIGHT ON THE NEXT WIRE.

The place value system makes it possible to write large numbers in shorthand. It also simplifies computation. This was a great advance over the Roman numerals used earlier. The place values in numbers are as follows.

1	One	
10	Ten	
100	Hundred	
1, 000	One	Thousand
10, 000	Ten	Thousand
100, 000	Hundred	Thousand
1, 000,000	One	Million
10, 000,000	Ten	Million
100, 000,000	Hundred	Million
1, 000,000,000	One	Billion
10, 000,000,000	Ten	Billion
100, 000,000,000	Hundred	Billion
1, 000,000,000,000	One	Trillion
10, 000,000,000,000	Ten	Trillion
100, 000,000,000,000	Hundred	Trillion

Note the repeating pattern of “one, ten, hundred” above. The first group of “one, ten, hundred” is the **Basic Group**. Next, we have the group of **Thousands**. Beyond that we have groups of *Millions, Billions, Trillions, Quadrillion, Quintillion, Sextillion, Septillion, Octillion, Nonillion, Decillion*, etc.

To develop skill with numbers one may use fingers at first, and then move to the next step of abacus. The use of abacus helps one visually see the system of place values. The next level is mental math where one learns to think systematically with numbers, assisted by paper and pencil, and calculators.

We feel strongly that student should first learn to do mental math before using calculator as an aid. Sole dependence on calculators and flash cards would prevent the student from developing the ability to think systematically. With the ability to think with numbers hampered, the student would not be able to learn math beyond the elementary level.

Today, we take this system for granted, but the brilliance of the concepts of zero, the digits, the Rule of Regrouping, and the place values is simply astounding when fully understood.

## GLOSSARY

<b>Abacus</b>	An <b>abacus</b> is a counting board with ten wires and ten beads on each wire. The word ABACUS comes from a word meaning, “a board sprinkled with dust for writing.” One can count up to billions on abacus.
<b>Arithmetic</b>	<b>Arithmetic</b> is the first aspect of Mathematics. The word ARITHMETIC comes from <i>arithmos</i> number + <i>techne</i> skill. <b>Arithmetic</b> literally means, “Skill with numbers.” It provides the skill needed to study <u>quantity</u> (not quality) of things.
<b>Basic group</b>	This refers to the <b>basic group</b> of place values: ONE, TEN, and HUNDRED. The basic group is followed by groups of thousand, Million, Billion, Trillion, Quadrillion, Quintillion, Sextillion, Septillion, Octillion, Nonillion, Decillion, etc. Each of these groups is made up of ONE, TEN, and HUNDRED.
<b>Billions</b>	This refers to the place values in the “ <b>Billion</b> ” group: ONE BILLION, TEN BILLION, and HUNDRED BILLION.
<b>Carry-over</b>	Whenever a count reaches 10 on a wire of an abacus, it is <b>carried over</b> as 1 on the next wire. Similarly, when adding numbers by columns, the “ten” of the sum in a column is carried over to the column on the left. See the RULE OF ABACUS.
<b>Counting</b>	The purpose of <b>counting</b> is to find out how many things are there. One <b>counts</b> by sequentially calling out for each item, one, two, three, four, five, and so on.
<b>Datum</b>	<b>Datum</b> is a single piece of information.
<b>Diagnostic test</b>	<b>Diagnostic Test</b> is a test to diagnose or analyze the understanding of the student.
<b>Digits</b>	The <b>digits</b> are symbols that we use to write numbers, much like letters are used to write words. For example, the number 386 is written with digits 3, 8, and 6. There are ten different digits – 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. These ten digits may be used to write all possible numbers.
<b>Expanded notation</b>	This is a notation in which a number is expressed in terms of its place values. See Lesson A1.13.
<b>Mathematics</b>	The word MATHEMATICS comes from a Greek word, <i>mathema</i> , which means, “Things learned.” Thus, <b>mathematics</b> consists of tools for learning. The purpose of Mathematics is to help develop the ability to think in a systematic manner. ABBREVIATION: Math or Maths.
<b>Mental math</b>	This is the third among the following gradients applied to the learning of arithmetic. (a) Counting on fingers (b) Counting on abacus (c) Mental math (d) Math with paper and pencil (e) Math with calculators.
<b>Milestone</b>	A <b>milestone</b> is a turning point. A “Math Milestone” refers to a turning point in the understanding of mathematics.

<b>Millions</b>	This refers to the place values in the “ <b>Million</b> ” group: ONE MILLION, TEN MILLION, and HUNDRED MILLION.
<b>Natural numbers</b>	The counting numbers are also referred to as <b>Natural numbers</b> . Zero is not a natural number because it is not used in counting.
<b>Number</b>	A <b>number</b> is a way of telling how many units there are. In counting, each count is given a different NUMBER, such as, one, two, three, and so on.
<b>Number base</b>	This is the base of the number because it determines how the number is to be constructed. It is the count at which regrouping occurs at any place in the number. Therefore, the largest digit used in the number is one less than this count.
<b>Place value</b>	<b>Place Value</b> is the value a digit gets from its place in a numeral. The place values in a numeral from right to left are: ONE, TEN, HUNDRED, THOUSAND, TEN THOUSAND, and so on.
<b>Quantity</b>	A <b>quantity</b> refers to “how many” or “how much” of something, as opposed to the description of that thing. A quantity describes the number of units.
<b>Regrouping, Rule of</b>	The <b>Rule of Regrouping</b> is, “When all beads are counted to the right on a wire, they are replaced by counting one bead to the right on the next wire.” This means, 10 ONES are equal to 1 TEN, 10 TENS are equal to 1 HUNDRED, 10 HUNDREDS are equal to 1 THOUSAND, and so on. This rule underlies the idea of carry-over.
<b>Systematic thinking</b>	When one gains familiarity with the fundamental ideas, which make up the subject of mathematics and thinks with them, then one can solve mathematical problems easily without resorting to memory. This is <b>systematic thinking</b> . Similar thinking may be developed for systems other than mathematics.
<b>Thousands</b>	This refers to the place values in the “ <b>Thousand</b> ” group: ONE THOUSAND, TEN THOUSAND, and HUNDRED THOUSAND.
<b>Unit</b>	The word UNIT means “one.” A <b>unit</b> is what we count one at a time to see “how many” or “how much” is there. When we count “one penny” at a time then each penny is a unit. When we count “ten pennies” at a time then each pile of “ten pennies” is a unit.
<b>Zero</b>	When there is no quantity, we call it zero. <b>Zero</b> is a placeholder for “absence of quantity.”