

## KINDERGARTEN MATH #5

# NUMBERS & OPERATIONS

*Numbers & Operations* introduces the basic operations with numbers and how such operations may be executed with skill.

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This is the fifth of the seven levels of the troubleshooting guide for KINDERGARTEN MATH. See *Summary* for details on all seven levels.

These lessons are designed for kindergarten, but they may be applied to anybody to fill earlier blanks in understanding.

***Start with the Diagnostic. If the diagnostic fails, then do the Lesson & Exercise.***

Follow these guidelines.

- (a) *When helping, make sure you have the attention of the student.*
- (b) *If you lose the attention, then go back to the point in the lesson where the student was attentive. Then come forward checking student's understanding.*
- (c) *Always approach any situation in an affectionate and relaxed manner.*
- (d) *Always encourage the student to ask questions.*
- (e) *Carefully listen to what the student has to say, and let the student know that you have heard him (or her).*
- (f) *Answer all questions matching the interest and understanding of the student.*

- (g) Always talk to the student at his (or her) level. Use only those terms and words that the student can easily understand.*
- (h) When teaching a new concept, ask the student to think examples of his own. Allow enough time even days to let that happen.*
- (i) Get the student involved and thinking with mathematical principles.*
- (j) In the final analysis, make sure that the student can apply mathematics with confidence.*

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Edited by Ivan Dorskocil

## DIAGNOSTICS & LESSONS

### ☺ Diagnostic K5.1      Cardinal Numbers

*To pass, the student should be able to tell that cardinal numbers indicate quantity.*

**"What cardinal number tells you how many days are in a week?"**

**"What are CARDINAL NUMBERS used for?"**

If the diagnostic fails, then do the Lesson & Exercise.

### Lesson & Exercise

*In this lesson the student learns that cardinal numbers indicate quantity.*

(a) Introduce the lesson.

**"In this lesson we are going to learn about cardinal numbers. A SET is a collection of things. A CARDINAL NUMBER tells you how many things there are in a set."**

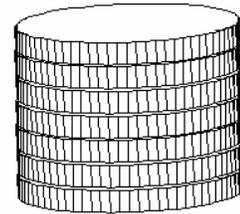
(b) Point to the following picture containing chairs.

**"This is a SET of 5 chairs. The number 5 tells you how many chairs are there in this set. Therefore, 5 is a cardinal number."**



(c) Place seven pennies in a pile.

**"There are 7 pennies in this pile, or set. Therefore, 7 is a cardinal number."**



(d) Place a foot ruler on the table.

**"There are 12 inches in a foot. Therefore, 12 is a cardinal number."**



(e) Explain.

**"A CARDINAL NUMBER tells you how many things are in a set. It indicates quantity. All counting numbers are cardinal numbers."**

- (f) Have the student indicate the cardinal numbers for
  1. The members in his or her family.
  2. The players in a baseball team
  3. The stars on the American flag
- (g) Have the student explain what CARDINAL NUMBERS are with examples.
- (h) Have the student ask you to explain cardinal numbers with examples.
- (i) Continue with this lesson until the student can confidently express the idea of cardinal numbers.
- (j) Repeat the diagnostic test.

## ☺ Diagnostic K5.2      Ordinal Numbers

*To pass, the student should be able to tell that ordinal numbers indicate position.*

**"What ordinal number tells you the position of the letter U in the word HOUSE?"**

**"What are ORDINAL NUMBERS used for?"**

If the diagnostic fails, then do the Lesson & Exercise.

### Lesson & Exercise

*In this lesson the student learns that ordinal numbers indicate position.*

(a) Start the lesson.

**"In this lesson we are going to learn about ordinal numbers. A SEQUENCE is how things follow one another. An ORDINAL NUMBER tells about the position in a sequence."**

(b) Show the following picture.

**"We have colors in a certain sequence in this picture."**



**"From the left pink is the FIRST color, red is the SECOND color, orange is the THIRD color, yellow is the FOURTH color, and so on."**

(c) Show the word HOUSE.

<b>H</b>	<b>O</b>	<b>U</b>	<b>S</b>	<b>E</b>
FIRST	SECOND	THIRD	FOURTH	FIFTH

**"The position of the letters in the word HOUSE is as follows:**

**'H' is the FIRST letter.**

**'O' is the SECOND letter.**

**'U' is the THIRD letter.**

**'S' is the FOURTH letter.**

**'E' is the FIFTH letter.**

**"The numbers FIRST, SECOND, THIRD, FOURTH, and FIFTH, indicate positions. These are ORDINAL numbers."**

(d) Have the student answer the following with respect to the word "MATHEMATICS"

1. What is the third letter?
2. In what position is the letter "E"?
3. What is the eighth letter?
4. In what position is the letter "C"?
5. How many letters are there in the word "MATHEMATICS"? Is this number CARDINAL or ORDINAL?

Answers: (1) T (2) Fifth (3) T (4) tenth (5) 11, CARDINAL

- (e) Have the student identify the underlined number as Cardinal or Ordinal”
1. There were ten students in the race.
  2. Michael came second in the race.
  3. John is the first batter in this inning.
  4. Each cricket team is made of eleven players.

Answers: (1) cardinal (2) ordinal (3) ordinal (4) cardinal

- (f) Have the student explain what ORDINAL NUMBERS are with examples.
- (g) Have the student ask you to explain ordinal numbers with examples.
- (h) Continue with this lesson until the student can confidently express the idea of ordinal numbers.
- (i) Repeat the diagnostic test.

## ☺ Diagnostic K5.3 Number Line

*To pass, the student should be able to grasp the relative position of numbers with the help of a number line.*

"Draw a NUMBER LINE and mark it from 0 to 10."

"Point to 7 and then to the next number?"

"Point to 4 and then to the previous number?"

"On the number line, does 6 come BEFORE 4, or AFTER?"

"Is the number 9 BETWEEN the numbers 3 and 7?"

"Is the number 7 CLOSER to 4 or 9?"

If the diagnostic fails, then do the Lesson & Exercise.

### Lesson & Exercise

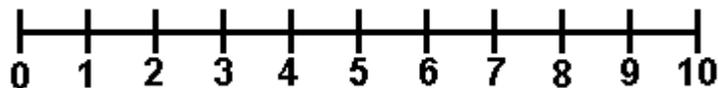
*In this lesson the student uses language such as before or after to describe relative position of whole numbers on a number line up to 10 or more.*

(a) Start the lesson.

"In this lesson we are going to learn about relative position of numbers with the help of a number line."

(b) Draw a number line.

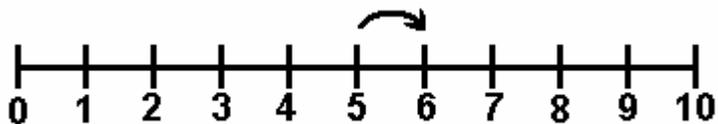
"This is a NUMBER LINE. The numbers follow each other as shown."



(c) Explain the idea of the NEXT number.

"As you count, you move to the right on the number line. The NEXT number is one more. For example, after 5 the NEXT number is 6 to the right."

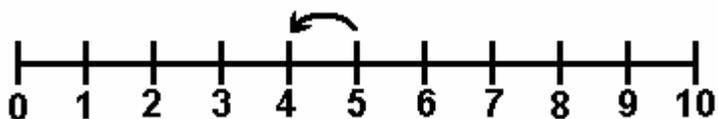
"From one number to the next, the distance is the same."



(d) Explain the idea of the PREVIOUS number.

"As you count backwards, you move to the left on the number line. The PREVIOUS number is one less. For example, before 5 the PREVIOUS number is 4 to the left."

"The next number comes AFTER, and the previous number comes BEFORE."

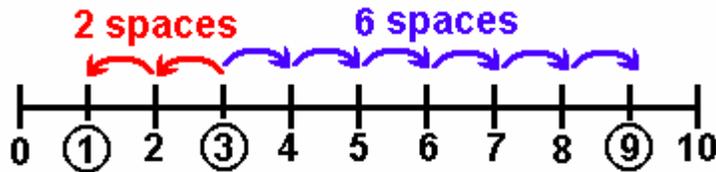


(e) Explain the position of a number relative to other numbers.

"On a number line we can easily see if a number is BETWEEN two other numbers. For example, we can see that 7 is BETWEEN 4 and 9."



“We can also see how far a number is from another. For example, on the number line below we can see that 3 is closer to 1 than to 9?”



- (f) Make piles of 1, 2, 3, 4, 5, 6, 7, etc., pennies on the table. Have the student arrange them in the order of increasing numbers.
- (g) Have the student draw a number line and mark it from 0 to 10.
- (h) Call out some numbers and have the student tell you the next number for each.
- (i) Call out some numbers and have the student give you the previous number for each.
- (j) Call out pair of numbers and have the student tell you which comes BEFORE, and which comes AFTER, on the number line.
- (k) Call out three numbers, and have the student tell you which number will be between the other two numbers on the number line. Have him ask you to do something similar.
- (l) Call out three numbers, and have the student tell you which two numbers are closer on the number line. Have him ask you to do something similar.
- (m) Have the student ask you to explain relative position of numbers with examples, such as, next, previous, before, after, etc..
- (n) Continue with this lesson until the student can confidently express the relative position of numbers.
- (o) Repeat the diagnostic test.

## ☺ Diagnostic K5.4 Equal to, More than, or Less than

To pass, the student should be able to compare two or more sets (up to 10 objects in each set) and identify which set is equal to, more than, or less than the other.

"Make two sets of pennies for each pair of numbers below."

(a) 5, 5                      (b) 5, 7                      (c) 5, 3

"Is the first set LESS than, EQUAL to, or MORE than, the second set?"

If the diagnostic fails, then do the Lesson & Exercise.

### Lesson & Exercise

In this lesson the student compares two or more sets (up to 10 objects in each set) and identifies which set is equal to, more than, or less than the other.

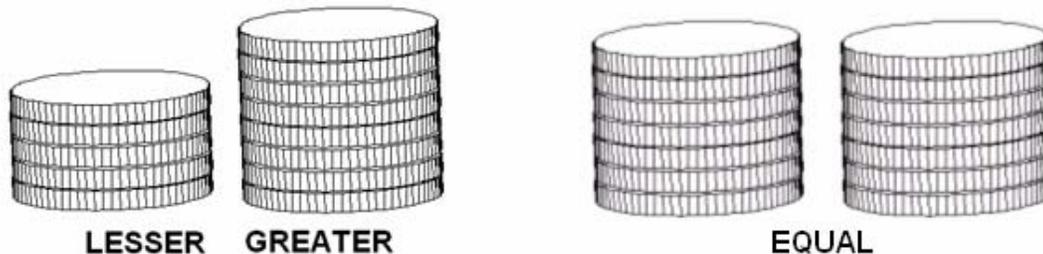
(a) Start the lesson.

"In this lesson we are going to compare one set to another.

(b) Make two piles of pennies.

"Here are two sets made up of 5, and 8 pennies respectively. We place these sets next to each other. Since they do not match penny for penny, they are not equal."

"The 8-penny set has more pennies than the 5-penny set. The 5-penny set is lesser and the 8-penny set is greater in comparison."



(c) Make another comparison.

"We place a 7-penny set next to another 7-penny set."

"The two sets match penny for penny. Therefore they are equal."

(d) Have the student make two sets per the following pair of numbers. Indicate if the first set is LESS than, EQUAL to, or MORE than, the second set.

(a) 5, 5                      (d) 5, 7                      (g) 10, 0  
(b) 3, 4                      (e) 2, 2                      (h) 6, 6  
(c) 2, 1                      (f) 8, 3                      (i) 0, 5

Answers: (1) equal (2) less (3) more (4) less (5) equal (6) more (7) more (8) equal (9) less

(e) Have the student ask you to make pair of sets and compare them.

(f) Continue with this lesson until the student can confidently compare two or more sets and identify which set is equal to, more than, or less than the other..

(g) Repeat the diagnostic test.

## ☺ Diagnostic K5.5 Equivalent forms of a Number

*To pass, the student should be able to represent equivalent forms of the same number, through the use of concrete materials, correctly and with confidence.*

**"Break a set of 10 pennies into smaller sets in three different ways."**

**"Fill in the blanks in three different ways to show the same number."**

(a)  $7 = \_ + \_ ,$       (b)  $7 = \_ + \_ ,$       (c)  $7 = \_ + \_$

If the diagnostic fails, then do the Lesson & Exercise.

### Lesson & Exercise

*In this lesson the student represents equivalent forms of the same number, up to 10 or more, through the use of concrete materials.*

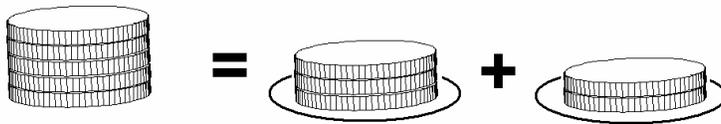
(a) Start the lesson.

**"In this lesson we are going to represent equivalent forms of the same number."**

(b) Place a pile of 5 pennies on the table.

**"Here is a set of 5 pennies."**

**"We can break this set into two sets of 3 and 2 pennies."**



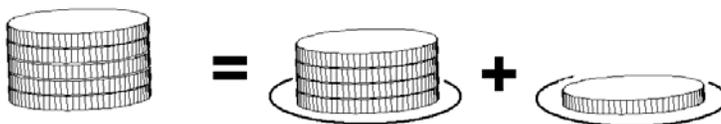
**"The symbol = means EQUAL TO. The symbol + means count together."**

**"The total number of pennies is still 5. Therefore, 5 pennies are the same as 3 and 2 pennies together."**

**" $3+2$  represents an equivalent form of 5."**

(c) Move one penny from the 2-penny pile to the 3-penny pile.

**"We now have 5 pennies broken into sets of 4 and 1 pennies."**



**"The total number of pennies is still 5. Therefore, 5 pennies are the same as 4 and 1 pennies together."**

**" $4+1$  represents an equivalent form of 5."**

(d) Place 5 pennies and a nickel on the table.

**"A nickel buys the same amount as 5 pennies."**



**"A NICKEL represents an equivalent form of 5 pennies."**

- (e) Place 10 pennies and a dime on the table.

**"A dime buys the same amount as 10 pennies."**



**"A DIME represents an equivalent form of 10 pennies."**

- (f) Have the student use coins to demonstrate that the number 7 may be written as, '1+6', '2+5,' and '3+4'. Have him ask you to do something similar.
- (g) Have the student demonstrate that 2 nickels may be shown as 1 nickel and 5 pennies.
- (h) Have the student demonstrate that a dime may be shown as 2 nickels.
- (i) Have the student use coins to figure out at least one equivalent form of the following numbers with non-zero numbers:
- |   |   |   |
|---|---|---|
| (a) $1 = \underline{0} + \underline{1}$         | (d) $4 = \underline{\quad} + \underline{\quad}$ | (g) $7 = \underline{\quad} + \underline{\quad}$ |
| (b) $2 = \underline{\quad} + \underline{\quad}$ | (e) $5 = \underline{\quad} + \underline{\quad}$ | (h) $8 = \underline{\quad} + \underline{\quad}$ |
| (c) $3 = \underline{\quad} + \underline{\quad}$ | (f) $6 = \underline{\quad} + \underline{\quad}$ | (i) $9 = \underline{\quad} + \underline{\quad}$ |
- (h) Have the student ask you to give equivalent form of some numbers.
- (i) Continue with this lesson until the student can confidently represent equivalent forms of the same number, through the use of concrete materials.
- (j) Repeat the diagnostic test.

## ☺ Diagnostic K5.6      Counting by 2's, 5's and 10's

*To pass, the student should be able to count orally to 100 or more by 1s, 2s, 5s, and 10s using a hundred chart or concrete materials.*

*"Count pennies ONE at a time to 100 or more."*

*"Count pennies TWO at a time to 100 or more."*

*"Count pennies FIVE at a time to 100 or more."*

*"Count pennies TEN at a time to 100 or more."*

If the diagnostic fails, then do the Lesson & Exercise.

### Lesson & Exercise

*In this lesson the student counts orally to 100 or more by 1s, 2s, 5s, and 10s using a hundred chart or concrete materials.*

#### HUNDREDS CHART

<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
<b>10</b>	<b>11</b>	<b>12</b>	<b>13</b>	<b>14</b>	<b>15</b>	<b>16</b>	<b>17</b>	<b>18</b>	<b>19</b>
<b>20</b>	<b>21</b>	<b>22</b>	<b>23</b>	<b>24</b>	<b>25</b>	<b>26</b>	<b>27</b>	<b>28</b>	<b>29</b>
<b>30</b>	<b>31</b>	<b>32</b>	<b>33</b>	<b>34</b>	<b>35</b>	<b>36</b>	<b>37</b>	<b>38</b>	<b>39</b>
<b>40</b>	<b>41</b>	<b>42</b>	<b>43</b>	<b>44</b>	<b>45</b>	<b>46</b>	<b>47</b>	<b>48</b>	<b>49</b>
<b>50</b>	<b>51</b>	<b>52</b>	<b>53</b>	<b>54</b>	<b>55</b>	<b>56</b>	<b>57</b>	<b>58</b>	<b>59</b>
<b>60</b>	<b>61</b>	<b>62</b>	<b>63</b>	<b>64</b>	<b>65</b>	<b>66</b>	<b>67</b>	<b>68</b>	<b>69</b>
<b>70</b>	<b>71</b>	<b>72</b>	<b>73</b>	<b>74</b>	<b>75</b>	<b>76</b>	<b>77</b>	<b>78</b>	<b>79</b>
<b>80</b>	<b>81</b>	<b>82</b>	<b>83</b>	<b>84</b>	<b>85</b>	<b>86</b>	<b>87</b>	<b>88</b>	<b>89</b>
<b>90</b>	<b>91</b>	<b>92</b>	<b>93</b>	<b>94</b>	<b>95</b>	<b>96</b>	<b>97</b>	<b>98</b>	<b>99</b>
<b>100</b>	<b>101</b>	<b>102</b>	<b>103</b>	<b>104</b>	<b>105</b>	<b>106</b>	<b>107</b>	<b>108</b>	<b>109</b>

(a) Start the lesson.

**"In this lesson we are going to count by 1's, 2's, 5's and 10's to 100 or more."**

- (b) Demonstrate counting by 1's by placing one penny on the table with each count.  
**"When counting by 1's we count 1 more each time starting from 0 like this... 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and so on."**
- (c) Demonstrate counting by 2s by placing two pennies on the table with each count.  
**"When counting by 2's we count 2 more each time starting from 0 like this... 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, and so on."**
- (d) Demonstrate counting by 5s by placing five pennies with each count.  
**"When counting by 5's we count 5 more each time starting from 0 like this... 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, and so on."**
- (e) Demonstrate counting by 10s by placing ten pennies with each count.  
**"When counting by 10's we count 10 more each time starting from 0 like this... 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, and so on."**
- (f) Have the student use coins to count orally to 100 or more by 1s, 2s, 5s, and 10s.
- (g) Have the student count orally to 100 or more by 1s, 2s, 5s, and 10s using a hundred's chart (see above).
- (h) Have the student count on ten fingers by 1s, 2s, 5s, and 10s.
- (i) Have the student ask you to count on ten fingers by 1s, 2s, 5s, and 10s.
- (j) Continue with this lesson until the student can confidently count orally to 100 or more by 1s, 2s, 5s, and 10s.
- (k) Repeat the diagnostic test.

## ☺ Diagnostic K5.7      Group Objects in Sets of 2 or More

*To pass, the student should be able to group objects in sets of 2s or more, correctly and with confidence.*

**"Group 60 pennies in sets of 2. How many sets are there?"**

**"Group 60 pennies in sets of 3. How many sets are there?"**

**"Group 60 pennies in sets of 4. How many sets are there?"**

**"Group 60 pennies in sets of 5. How many sets are there?"**

If the diagnostic fails, then do the Lesson & Exercise.

### Lesson & Exercise

*In this lesson the student groups objects in sets of 2 or more.*

(a) Start the lesson.

**"In this lesson we are going to learn to group objects."**

(b) Place 60 pennies on the table. Group them in sets of 2s, and count how many sets of 2s were created.

**"Let's group these pennies in sets of 2s. There are 30 sets of 2s."**



(c) Gather up all the pennies. Group them in sets of 3s, and count the number of sets.

**"Let's group these pennies in sets of 3s. There are 20 sets of 3s."**

(d) Gather up all the pennies. Group them in sets of 4s, and count the number of sets.

**"Let's group these pennies in sets of 4s. There are 15 sets of 4s."**

(e) Gather up all the pennies. Group them in sets of 5s.

**"Let's group these pennies in sets of 5s. There are 12 sets of 5s."**

(f) Gather up all the pennies. Group them in sets of 6s.

**"Let's group these pennies in sets of 6s. There are 10 sets of 6s."**

(g) Gather up all the pennies. Group them in sets of 10s.

**"Let's group these pennies in sets of 10s. There are 6 sets of 10s."**

(h) Have the student group 60 pennies in sets of 2, 3, 4, 5, 6, and 10 respectively.

(i) Have the student ask you to group 60 pennies in sets of 2, 3, 4, 5, 6, and 10 respectively.

(j) Continue with this lesson until the student can confidently group objects in sets of 2 or more.

(k) Repeat the diagnostic test.

## ☺ Diagnostic K5.8 Larger and Smaller Numbers

*To pass, the student should be able to show the relationships between larger and smaller numbers, correctly and with confidence.*

"Circle the larger number and underline the smaller number. Use number line as necessary.

- (a) 9, 5                      (b) 5, 8                      (c) 3, 2

If the diagnostic fails, then do the Lesson & Exercise.

### Lesson & Exercise

*In this lesson the student looks at the relationships between larger numbers and smaller numbers.*

- (a) Start the lesson.

"In this lesson we are going to look at the relationships between larger numbers and smaller numbers."

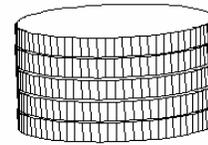
- (b) Explain the relationship of a number to a set.

"A number tells you how many things are there in a set.

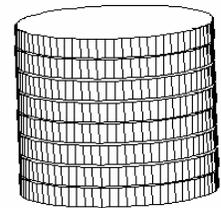
"For sets of pennies, a larger set will have larger number of pennies. A smaller set will have smaller number of pennies.

"Let's compare a 5-penny set to an 8-penny set.

"The 8-penny set is larger. The 5-penny set is smaller. Therefore, the number 8 is larger than the number 5. You have to count 3 more from 5 to get to 8."



LESSER



GREATER

- (c) Explain the relationship on the number line.



"This relationship can be observed easily on a number line. The larger numbers appear to the right of the smaller numbers. The smaller numbers appear to the left of the larger numbers."

- (d) Among the following pairs of numbers, have the student circle the larger number and underline the smaller number. Check it on a number line as necessary.

- |          |          |           |
|----------|----------|-----------|
| (1) 3, 5 | (4) 5, 7 | (7) 0, 10 |
| (2) 3, 2 | (5) 4, 2 | (8) 6, 5  |
| (3) 2, 1 | (6) 8, 3 | (9) 0, 5  |

Answers: The larger numbers are: (1) 5 (2) 3 (3) 2 (4) 7 (5) 4 (6) 8 (7) 10 (8) 6 (9) 5

- (e) Have the student ask you to show relationship between larger and smaller numbers.  
 (f) Continue with this lesson until the student can confidently explain relationships between larger numbers and smaller numbers.  
 (g) Repeat the diagnostic test.

## ☺ Diagnostic K5.9      Putting Sets together

*To pass, the student should be able to demonstrate and describe the effect of putting together sets of objects.*

**"Describe the effect of putting together the following sets of objects."**

- (1) 4 pennies and 2 pennies
- (2) 3 fingers and 4 fingers
- (3) 5 pennies and 3 pennies

If the diagnostic fails, then do the Lesson & Exercise.

### Lesson & Exercise

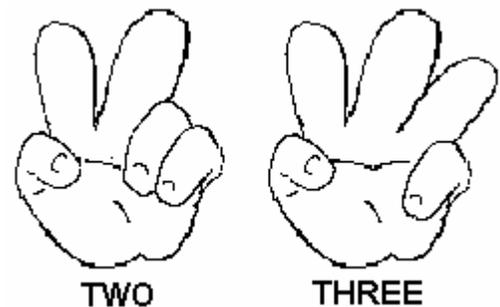
*In this lesson the student demonstrates and describes the effect of putting together sets of objects (for example, 3 cubes and 4 cubes is 7 cubes).*

(a) Start the lesson.

**"In this lesson we are going to demonstrate and describes the effect of putting together sets of objects."**

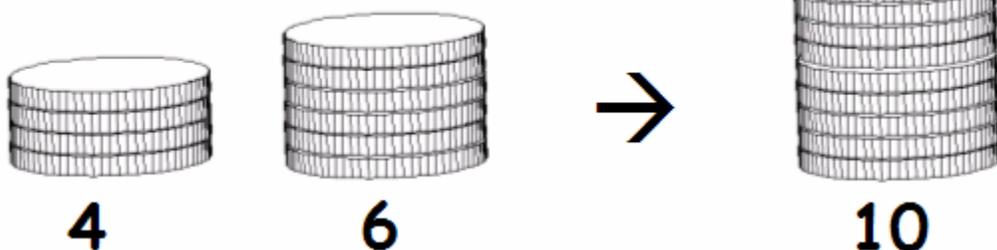
(b) Raise 2 fingers on one hand. Raise 3 fingers on the other hand. Then put them together.

**"Here are 2 fingers on one hand and 3 fingers on the other hand. "When we put them together we get 5 fingers."**



(c) Place 4 and 6 pennies in two piles. Then put them together in one pile.

**"Here we have a set of 4 pennies and another set of 6 pennies. When we put them together we get a set of 10 pennies."**



(d) Explain the effect.

**"The effect of putting together sets of objects is that we get a SUM of objects. This process of getting a sum is called ADDITION."**

(e) Demonstrate addition on abacus.

**"Let's add 5 and 3 on abacus. We start with all the beads to the left. We first move 5 beads to the right on the first wire, and then move 3 more beads."**



"There are 8 beads on the right. This is the sum."



(f) Write this down using math symbols.

"We may write this addition as follows.

$$5 + 3 = 8$$

"We read this as, 5 plus 3 equals 8."

(g) Have the student describe the effect of putting together the following sets of objects.

- |                             |                             |
|-----------------------------|-----------------------------|
| (1) 1 finger and 2 fingers  | (4) 4 pennies and 2 pennies |
| (2) 2 fingers and 4 fingers | (5) 5 pennies and 3 pennies |
| (3) 4 fingers and 3 fingers | (6) 3 pennies and 3 pennies |

(h) Have the student describe the effect of putting together the following sets of pennies.

- |                                 |                                  |                                  |
|---------------------------------|----------------------------------|----------------------------------|
| (1) $1 + 1 = \underline{\quad}$ | (6) $6 + 1 = \underline{\quad}$  | (11) $2 + 2 = \underline{\quad}$ |
| (2) $2 + 1 = \underline{\quad}$ | (7) $7 + 1 = \underline{\quad}$  | (12) $3 + 2 = \underline{\quad}$ |
| (3) $3 + 1 = \underline{\quad}$ | (8) $8 + 1 = \underline{\quad}$  | (13) $4 + 2 = \underline{\quad}$ |
| (4) $4 + 1 = \underline{\quad}$ | (9) $9 + 1 = \underline{\quad}$  | (14) $5 + 2 = \underline{\quad}$ |
| (5) $5 + 1 = \underline{\quad}$ | (10) $1 + 2 = \underline{\quad}$ | (15) $6 + 2 = \underline{\quad}$ |

(i) Have the student add the following sets of beads on abacus, and describe the sum:

- |                                 |                                  |                                  |
|---------------------------------|----------------------------------|----------------------------------|
| (1) $7 + 2 = \underline{\quad}$ | (6) $4 + 3 = \underline{\quad}$  | (11) $2 + 4 = \underline{\quad}$ |
| (2) $8 + 2 = \underline{\quad}$ | (7) $5 + 3 = \underline{\quad}$  | (12) $3 + 4 = \underline{\quad}$ |
| (3) $1 + 3 = \underline{\quad}$ | (8) $6 + 3 = \underline{\quad}$  | (13) $4 + 4 = \underline{\quad}$ |
| (4) $2 + 3 = \underline{\quad}$ | (9) $7 + 3 = \underline{\quad}$  | (14) $5 + 4 = \underline{\quad}$ |
| (5) $3 + 3 = \underline{\quad}$ | (10) $1 + 4 = \underline{\quad}$ | (15) $6 + 4 = \underline{\quad}$ |

(j) Have the student add the following numbers, and write the sum.

- |                                 |                                  |                                  |
|---------------------------------|----------------------------------|----------------------------------|
| (1) $1 + 5 = \underline{\quad}$ | (6) $1 + 6 = \underline{\quad}$  | (11) $2 + 7 = \underline{\quad}$ |
| (2) $2 + 5 = \underline{\quad}$ | (7) $2 + 6 = \underline{\quad}$  | (12) $3 + 7 = \underline{\quad}$ |
| (3) $3 + 5 = \underline{\quad}$ | (8) $3 + 6 = \underline{\quad}$  | (13) $1 + 8 = \underline{\quad}$ |
| (4) $4 + 5 = \underline{\quad}$ | (9) $4 + 6 = \underline{\quad}$  | (14) $2 + 8 = \underline{\quad}$ |
| (5) $5 + 5 = \underline{\quad}$ | (10) $1 + 7 = \underline{\quad}$ | (15) $1 + 9 = \underline{\quad}$ |

(k) Call out any two numbers less than ten. Have the student count that many fingers or pennies together, and give you the sum.

(l) Have the student ask you to do something similar.

(m) Continue with this lesson until the student can confidently describe the effect of putting together sets of objects.

(n) Repeat the diagnostic test.

## ☺ Diagnostic K5.10 Addition with Numbers

*To pass, the student should be able to solve addition word problems (with numbers that add up to not more than 10) correctly, and with confidence, three times in a row.*

**"Answer the following questions."**

- (a) Lisa caught 3 butterflies yesterday and 5 more today. How many butterflies did she catch in all?
- (b) Billy has 7 marbles. He collected 2 more marbles. How many marbles does Billy have?
- (c) Elli had 5 dolls. She got 4 new dolls on her birthday. How many dolls does Elli have?

If the diagnostic fails, then do the Lesson & Exercise.

### Lesson & Exercise

*In this lesson the student demonstrates an awareness of addition in everyday activities (using concrete objects, models, drawings, role playing).*

- (a) Start the lesson.  
**"In this lesson we are going to introduce addition in everyday activities."**
- (b) State the following problem while drawing as many dogs as the numbers.  
**"Suppose Johnny has 2 dogs, and Mary has 5 dogs. How many dogs do Johnny and Mary have together?"**
- (c) Simplify the problem.  
**"We can ask this problem more simply as, 'what are 2 dogs and 5 dogs?'"**
- (d) Write the problem using addition symbol.  
**"We may write this question as,  $2 \text{ dogs} + 5 \text{ dogs} = \text{How many dogs?}$   
Here '+' (plus) means that we have to count the dogs together."**
- (e) State the problem more simply.  
**"We may write this problem as,  $2 + 5 = \underline{\quad}$  (dogs)?"**
- (f) Solve the problem.  
**"We get the answer by counting 2 and 5 together,  $2 + 5 = 7$  (dogs)?  
"Therefore, Johnny and Mary have 7 dogs together."**
- (g) Show how to translate a problem from English to Math.  
**"We translate addition problems from English to Math like this.**  
**ENGLISH: 2 fingers and 4 fingers are equal to 6 fingers.**  
**MATH:  $2 + 4 = 6$  (fingers)**  
**The idea of putting together translates as '+' (plus)."**
- (h) Have the student translate the following into the language of mathematics:
  - (1) 4 chairs and 4 chairs are equal to 8 chairs.
  - (2) 6 pennies and 4 pennies are equal to 10 pennies.
  - (3) 8 houses and 3 houses are equal to 11 houses.

- (i) Have the student explain what the following means:
- (1)  $3 + 3 = 6$  (fingers)
  - (2)  $2 + 1 = 3$  (cats)
  - (3)  $5 + 4 = 9$  (dogs)
- (j) Have the student translate and solve the following word problems:
- (1) Johnny ate 1 candy in the morning and 2 candies in the afternoon. How many candies did he eat that day?
  - (2) Joan saved 4 quarters last week and 1 quarter this week. How many quarters does she have?
  - (3) For his birthday, Peter got 3 books from his uncle and 3 books from his aunt. How many books did Peter receive on his birthday?
  - (4) On Christmas, Lisa lit 4 candles and Tommy also lit 4 candles. How many candles did they light altogether?
  - (5) You caught 3 butterflies on Monday and 5 butterflies on Tuesday. How many butterflies did you catch in all?
  - (6) Billy has 7 marbles. He collected 2 more. How many marbles does he have?
  - (7) Elli had 5 dolls. She got 4 more on her birthday. How many dolls does Elli have?
  - (8) Joan had 6 quarters. She received 3 more quarters from her dad. How many quarters does Joan have?
  - (9) Johnny jumped forward 3 feet and then 4 feet. What is the total distance that he jumped?
  - (10) Bobby read 2 books last week and 4 books this week. How many books did Bobby read?
- (k) Have the student ask you to do something similar.
- (l) Continue with this lesson until the student can demonstrate an awareness of addition in everyday activities.
- (m) Repeat the diagnostic test.

## ☺ Diagnostic K5.11 Taking Sets apart

*To pass, the student should be able to demonstrate and describe the effect of taking apart sets of objects.*

"Describe the effect of taking apart the following sets of objects.

- (1) 2 pennies from 3 pennies
- (2) 2 fingers from 5 fingers
- (3) 5 pennies from 7 pennies

If the diagnostic fails, then do the Lesson & Exercise.

### Lesson & Exercise

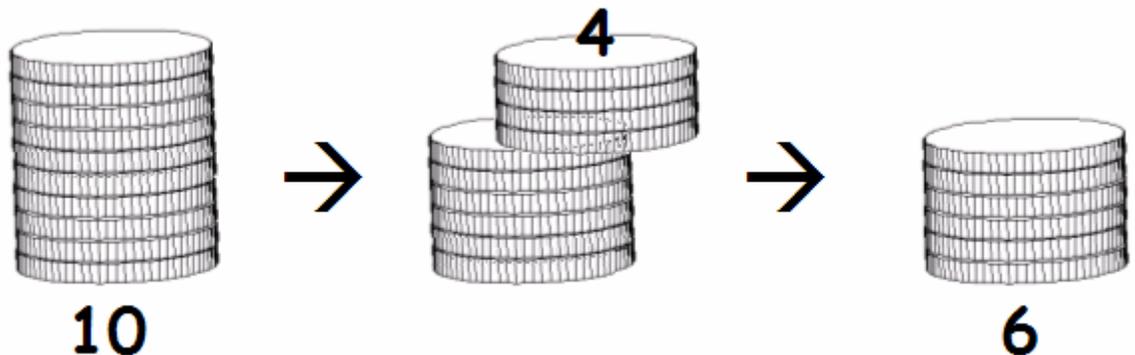
*In this lesson the student demonstrates and describes the effect of taking apart sets of objects.*

(a) Start the lesson.

"In this lesson we are going to demonstrate and describe the effect of taking apart sets of objects."

(b) Raise 5 fingers on one hand. Hide 2 fingers.

"Here we have 5 fingers. Let's hide 2 fingers. We have 3 fingers left."



(c) Place 10 pennies on the table. Take apart 4 pennies.

"Here we have 10 pennies. Let's take apart 4 pennies. There are 6 pennies left."

(d) Explain the effect.

"The effect of taking apart sets of objects is that we get a REMAINDER of objects. This process of getting a remainder is called SUBTRACTION."

(e) Demonstrate subtraction on abacus.

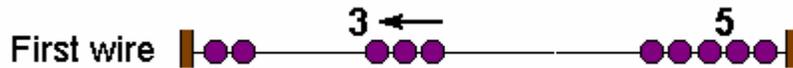
"Let's take apart 3 from 8 on abacus."

(f) Start with the zero position. Move 8 beads to the right.

"We have 8 beads to the right on the first wire."



"Then we take apart 3 beads by moving them back."



"We count the beads on the right. The remainder is 5 beads."

(g) Write this down using math symbols.

"We may write this process of subtraction as,

$$8 - 3 = 5$$

"We read this as, 8 minus 3 equals 5."

(h) Have the student describe the effect of taking apart the following sets of objects.

- |                              |                              |
|------------------------------|------------------------------|
| (1) 2 pennies from 3 pennies | (4) 2 pennies from 6 pennies |
| (2) 3 pennies from 3 pennies | (5) 1 penny from 6 pennies   |
| (3) 5 pennies from 7 pennies | (6) 3 pennies from 8 pennies |

(i) Have the student describe the effect of taking apart the following sets of objects.

- |                                 |                                  |                                  |
|---------------------------------|----------------------------------|----------------------------------|
| (1) $9 - 3 = \underline{\quad}$ | (6) $6 - 5 = \underline{\quad}$  | (11) $6 - 6 = \underline{\quad}$ |
| (2) $8 - 5 = \underline{\quad}$ | (7) $7 - 7 = \underline{\quad}$  | (12) $7 - 3 = \underline{\quad}$ |
| (3) $6 - 2 = \underline{\quad}$ | (8) $8 - 1 = \underline{\quad}$  | (13) $9 - 5 = \underline{\quad}$ |
| (4) $9 - 1 = \underline{\quad}$ | (9) $6 - 4 = \underline{\quad}$  | (14) $8 - 3 = \underline{\quad}$ |
| (5) $8 - 2 = \underline{\quad}$ | (10) $5 - 5 = \underline{\quad}$ | (15) $7 - 4 = \underline{\quad}$ |

(j) Have the student subtract the following numbers, and write the remainder:

- |                                 |                                  |                                  |
|---------------------------------|----------------------------------|----------------------------------|
| (1) $7 - 3 = \underline{\quad}$ | (6) $9 - 2 = \underline{\quad}$  | (11) $6 - 5 = \underline{\quad}$ |
| (2) $8 - 4 = \underline{\quad}$ | (7) $8 - 5 = \underline{\quad}$  | (12) $4 - 2 = \underline{\quad}$ |
| (3) $7 - 5 = \underline{\quad}$ | (8) $7 - 6 = \underline{\quad}$  | (13) $3 - 3 = \underline{\quad}$ |
| (4) $9 - 4 = \underline{\quad}$ | (9) $5 - 1 = \underline{\quad}$  | (14) $8 - 6 = \underline{\quad}$ |
| (5) $9 - 6 = \underline{\quad}$ | (10) $8 - 7 = \underline{\quad}$ | (15) $9 - 0 = \underline{\quad}$ |

(k) Call out two numbers less than ten. Have the student use fingers or pennies to take apart the smaller number from the larger number, and state the remainder.

(l) Have the student ask you to do something similar.

(m) Continue with this lesson until the student can describe the effect of taking apart sets of objects.

(n) Repeat the diagnostic test.

## ☺ Diagnostic K5.12 Subtraction with Numbers

*To pass, the student should be able to solve subtraction word problems (with numbers not more than 10) correctly, and with confidence, three times in a row.*

**"Answer the following questions."**

- (a) Amy has 7 kittens. She gave away 3. How many kittens are left?
- (b) Johnny had 5 candies. He ate 3 of them. How many candies does he have now?
- (c) John has 9 hamsters. 5 of them ran away. How many hamsters are left?

If the diagnostic fails, then do the Lesson & Exercise.

### Lesson & Exercise

*In this lesson the student demonstrates an awareness of subtraction in everyday activities (using concrete objects, models, drawings, role playing).*

- (a) Start the lesson.  
**"In this lesson we are going to introduce subtraction in everyday activities."**
- (b) State the following problem while drawing as many chairs.  
**"If we remove 2 chairs from 6 chairs, how many chairs are left?"**
- (c) Simplify the problem.  
**"We can ask this problem more simply as this."  
"What is taking apart 2 chairs from 6 chairs?"**
- (d) Write the problem using subtraction symbol.  
**"We may write this question as,        6 chairs – 2 chairs    =    How many chairs?  
Here '-' (minus) means that we have to take apart as many chairs as follow this symbol."**
- (e) State the problem more simply.  
**"We may write this problem as,        6 – 2    =    \_\_\_    (chairs)?"**
- (f) Solve the problem.  
**"We get the answer by taking apart 2 from 6,        6 - 2    =    4    (chairs)  
"Therefore, we have 4 chairs left."**
- (g) Show how to translate a problem from English to Math.  
**"We translate subtraction problems from English to Math like this."  
  
ENGLISH:    If you have 5 dollars, and you spend 2 dollars, you are left with 3 dollars."  
MATH:        5 - 2 = 3        (pennies)  
  
"We translate the idea of taking apart as '-' (minus)."**
- (h) Have the student translate the following into the language of mathematics:
  - (1) Taking apart 4 fingers from 6 fingers is equal to 2 fingers left.
  - (2) Taking apart 3 pennies from 9 pennies is equal to 6 pennies left.
  - (3) Taking apart 2 pennies from 7 pennies is equal to 5 pennies left.
- (i) Have the student explain what the following means:

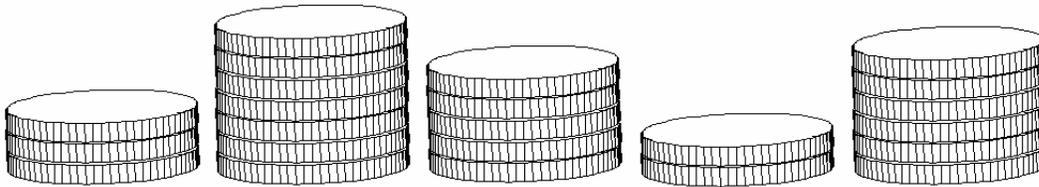
- (1)  $2 - 1 = 1$  (finger)
- (2)  $6 - 4 = 2$  (fingers)
- (3)  $4 - 1 = 3$  (pennies)

- (j) Have the student translate and solve the following word problems:
  - (1) Amy has 7 kittens. She gave away 3. How many kittens are left?
  - (2) John has 9 hamsters. 5 of them ran away. How many hamsters are left?
  - (3) Bill had collected 7 pennies. He lost 7 pennies. How many pennies is he left with?
  - (4) Johnny had 5 candies. He ate 3 of them. How many candies does he have now?
  - (5) Joan had saved 10 quarters. She spent 6 of them on an ice-cream cone. How many quarters is she left with?
  - (6) Billy caught 9 butterflies during the week. 4 of these flew away. How many butterflies were left?
- (k) Have the student ask you to do something similar.
- (l) Continue with this lesson until the student can describes the effect of taking apart sets of objects.
- (m) Repeat the diagnostic test.

## ☺ Diagnostic K5.13 Estimate Numbers

*To pass, the student should be able to estimate and compare sets containing different number of objects.*

**"Estimate the number of pennies in each of the sets below."**



**"Rearrange the sets from fewest to most numbers by comparing your estimates?"**

If the diagnostic fails, then do the Lesson & Exercise.

### Lesson & Exercise

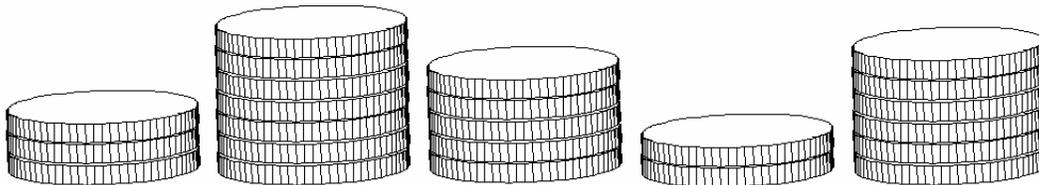
*In this lesson the student estimates and verifies by counting sets that have more, fewer, or the same number of objects.*

(a) Start the lesson.

**"In this lesson we are going to estimate sets that have more, fewer, or the same number of objects."**

(b) Set up piles of pennies as in the picture below.

**"There are 5 piles of pennies on the table. Each pile is a SET of pennies. Therefore, we have 5 SETS of pennies on the table."**



**"You may estimate the size of each set by the height of the stack. Let's rearrange these sets by their height."**



(c) Count the pennies in each set.

**"The pennies in these sets are 2, 3, 5, 6, and 7 respectively. Therefore, our estimate of which set has fewer and which has more numbers is correct."**

- (d) Have the student make several piles of pennies, such that, some of them are equal and others not.
- (e) Have the student estimate the number of pennies in these sets and rearrange them in the sequence of fewest to the most, and then verify the estimate by counting. Have him ask you to do the same.
- (f) Have the student arrange the following words from the shortest to the longest by estimating the number of letters used to construct them, and then verify the estimate by counting.

**MATHEMATICS, UNIT, ABACUS, COUNT, NUMBER, MEASURE, TEN, EIGHT**

Have him ask you to do the same.

- (g) Continue with this lesson until the student can describe the effect of taking apart sets of objects.
- (h) Repeat the diagnostic test.

## ☺ Diagnostic K5.14    **ODD & Even Numbers**

*To pass, the student should be able to build models to show that numbers are either odd or even (up to 10), with confidence*

**"Circle the ODD numbers. Underline the EVEN numbers."**

**9   5   4   8   1   6   3   7   2**

If the diagnostic fails, then do the Lesson & Exercise.

### **Lesson & Exercise**

*In this lesson, the student builds models to show that numbers are either odd or even (up to 10).*

(a) Start the lesson.

**"In this lesson we are going to introduce EVEN and ODD numbers."**

(b) Place 6 pennies on the table.

**"Here are 6 pennies. I am going to pair them up in sets of 2's."**



(c) Pair up the pennies.

**"Note that all pennies are paired up."**

(d) Explain the EVEN number.

**"When a number of pennies can be paired up, then you have an EVEN number of pennies. 6 is an EVEN number because 6 pennies can be paired up."**

(e) Place 5 pennies on the table.

**"Here are 5 pennies. I am going to pair them up in sets of 2's."**



(f) Pair up the pennies.

**"Note that one penny is left unpaired."**

(g) Explain the ODD number.

**"When pairing a number of pennies, a penny is left unpaired, then you have an ODD number of pennies. 5 is an ODD number because when you pair up 5 pennies, a penny is left unpaired."**

(h) Write the numbers up to 10 alternately in two columns.

**"Let's write the numbers alternately in two columns."**

FIRST	SECOND
1	2
3	4
5	6
7	8
9	10

- (i) Explain the columns.  
**"The First column contains the ODD numbers 1, 3, 5, 7, and 9.**  
**"The Second column contains the EVEN numbers 2, 4, 6, 8, and 10."**
- (j) Have the student write the numbers up to 20 alternately in two columns, and identify the column of even numbers, and the column of odd numbers.
- (k) Have the student write at least three even numbers.
- (l) Have the student write at least three odd numbers.
- (m) Continue with this lesson until the student can confidently identify even and odd numbers.
- (n) Repeat the diagnostic test.

## ☺ Diagnostic K5.15    Comparing values

*To pass, the student should be able to compare the values of a penny, nickel, and dime correctly, and with confidence.*

**"How many pennies are equal in value to a nickel?"**

**"How many pennies are equal in value to a dime?"**

**"How many nickels are equal in value to a dime?"**

If the diagnostic fails, then do the Lesson & Exercise.

### Lesson & Exercise

*In this lesson the student learns and compares the values of a penny (1 cent), nickel (5 cents), and dime (10 cents).*

(a) Start the lesson.

**"In this lesson we are going to compare the values of a penny (1 cent), nickel (5 cents), and dime (10 cents)."**

(b) Place a penny on the table.

**"This is a penny. Its value is 1 cent. You may buy 1 sweet with it."**



(c) Place a nickel on the table.

**"This is a nickel. Its value is 5 cents. You may buy 5 sweets with it."**

**"Therefore, a nickel is worth 5 pennies."**



(d) Place a dime on the table.

**"This is a dime. Its value is 10 cents. You may buy 10 sweets with it."**

**"Therefore, a dime is worth 10 pennies."**



**"Each of the 5 pennies above may be replaced by a nickel."**

**"Therefore, a dime is worth 2 nickels."**

- (e) Have the student convert pennies to nickels as follows:
  - (1) Place 25 pennies on the table.
  - (2) Make as many 5-pennies stacks out of them as possible.
  - (3) Replace each 5-pennies stack by a nickel.
  
- (f) Have the student convert pennies to dimes as follows:
  - (1) Place 40 pennies on the table.
  - (2) Make as many 10-pennies stacks out of them as possible.
  - (3) Replace each 10-pennies stack by a dime.
  
- (g) Have the student convert nickels to pennies, and pennies to as many nickels as possible.
  
- (h) Have the student convert dimes to pennies, and pennies to as many dimes as possible.
  
- (i) Continue with this lesson until the student can confidently group objects in sets of 2 or more.
  
- (j) Repeat the diagnostic test.

## SUMMARY

This is the fifth of the seven levels of the Troubleshooting Guide for KINDERGARTEN MATH. The Troubleshooting Guide for Kindergarten introduces the concept of UNIT, and explores ways to measure length, weight, capacity and time. It further develops the concept of counting into the concepts of addition and subtraction.

The Kindergarten troubleshooting guide is divided into the following levels:

### **(1) ORIENTATION & SPATIAL SENSE**

Orientation and Spatial Sense forms the foundation of the subject of GEOMETRY. It introduces the elements of space and how they relate to us.

### **(2) NUMBERS & PLACE VALUES**

Numbers and Place Values form the foundation of the subject of ARITHMETIC. It introduces a system of whole numbers to represent quantities in a simple manner.

### **(3) UNITS & FRACTIONS**

Units & Fractions addresses ways to represent quantities, which cannot be represented by whole numbers.

### **(4) COUNTING & MEASUREMENTS**

Counting & Measurements provides ways to determine the various magnitudes. It helps to bring familiarization with the use of numbers.

### **(5) NUMBERS & OPERATIONS**

Numbers & Operations introduces the basic operations with numbers and how such operations may be executed with skill.

### **(6) PATTERNS & RELATIONAL SENSE**

Patterns and Relational Sense forms the foundation of the subject of ALGEBRA. It is a study of patterns underlying numbers, and quantitative relationships.

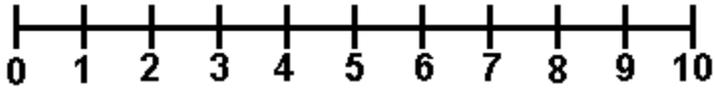
### **(7) DATA ANALYSIS & PROBABILITY**

Data Analysis & Probability shows how to display quantitative comparisons graphically. It introduces the estimation of likelihood.

Though these lessons are designed for the kindergarten level, these diagnostic actions may be used for students at higher grades to help discover and resolve missing basics.

## GLOSSARY

[For additional words refer to the glossary at the end of earlier levels.]

<b>Addition</b>	ADDITION is counting the items of two sets together.
<b>Cardinal</b>	A CARDINAL NUMBER tells you how many things there are in a set. For example, in a set describing 4 chairs, 4 is a cardinal number.
<b>Equivalent Form</b>	"3+2" and "5" are the equivalent forms of the same number.
<b>Even</b>	When all items in a set can be paired up with none left, then we have EVEN number of items.
<b>Number Line</b>	A NUMBER LINE is a line divided into equal spaces. These spaces are the numbered in a sequence.  A horizontal number line with vertical tick marks at each integer from 0 to 10. The numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 are printed below their respective tick marks.
<b>Odd</b>	When an item is left unpaired after pairing up rest of the items in a set, then we have ODD number of items.
<b>Operation</b>	An OPERATION is an action done on numbers such as addition and subtraction.
<b>Ordinal</b>	An ORDINAL NUMBER describes the position in a sequence. For example, <i>third</i> describes the position of letter "U" in the word HOUSE. Therefore, "third" is an ordinal number.
<b>Sets</b>	A SET is a collection of things. For example, there may be a set of 4 chairs around a table.
<b>Sequence</b>	A SEQUENCE is how things follow one another. For example, the months in a year follow a certain sequence.
<b>Subtraction</b>	SUBTRACTION is taking one set apart from another.
<b>Value</b>	A "nickel" and "5 pennies" have the same value in terms of what they can buy.